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Acoustic Impedance and Sound Absorption

PHILIP M. MORSE, RICHARD H. BOLT* AND RICHARD L. BROWN
Eastman Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts

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Theoretical curves for acoustic impedance *vs.* frequency, obtained by solving the equation for wave motion within the material, are compared with recently measured values of the impedance of various sound absorbing materials. The satisfactory agreement in most cases measured indicates that that for homogeneous materials the effective values of flow resistance, porosity and material density are nearly constant over the frequency range 200 to 6000 c.p.s. This makes it possible to express the whole acoustic behavior of these materials by three constants. The behavior of two nonhomogeneous materials is also discussed. The relation between impedance and the various types of absorption coefficient is discussed, and specific formulas for the coefficients are given in terms of the acoustic impedance. It is indicated that in rectangular chambers, when the sound does not have a random diffusion throughout the room, the decay curve for pressure squared in db, *vs.* time in seconds, is a broken line, with the initial slope corresponding to a coefficient for normal incidence, and the final slope corresponding to a coefficient for grazing incidence. If enough scattering objects are inserted to make the diffusion of sound complete, so that no standing wave is all grazing or all normal to any surface, then the decay curve is a straight line with slope corresponding to the coefficient defined by Sabine. A formula relating this coefficient to the acoustic impedance is given. The Acoustical Materials Association values of absorption coefficient, used in practice, are compared with the curves of normal and Sabine coefficient, computed from the experimental curves for impedance. It is apparent that above 2000 c.p.s. the A.M.A. values equal the computed Sabine coefficients, indicating sufficient diffusion of sound in the reverberation chambers at these frequencies. Below 500 c.p.s. the A.M.A. values correspond more closely to the normal coefficient, indicating an insufficient diffusion of sound, and that the initial slope of a broken line decay curve is being measured. In these two ranges of frequency, therefore, one can compute the A.M.A. coefficients from the impedance. It is suggested that field coefficients for large auditoriums correspond to the computed Sabine coefficient for lower frequencies than do the A.M.A. coefficients. A simple formula is given for the broken line decay curve in a rectangular room with no diffusion, and is checked by experimental curves. A case of partial diffusion is discussed, where the decay curve is intermediate between the theoretical curves for complete and for no diffusion.

THE excellent measurements by Beranek[1] of the acoustic impedance of various commercial acoustic materials now make it possible to demonstrate the adequacy of the theoretical connection between the acoustic impedance of the material and its physical properties on the one hand, and between its impedance and the absorption coefficient as employed in usual

* National Research Fellow in Physics.
[1] L. L. Beranek, J. Acous. Soc. Am. **12**, 14 (1940).

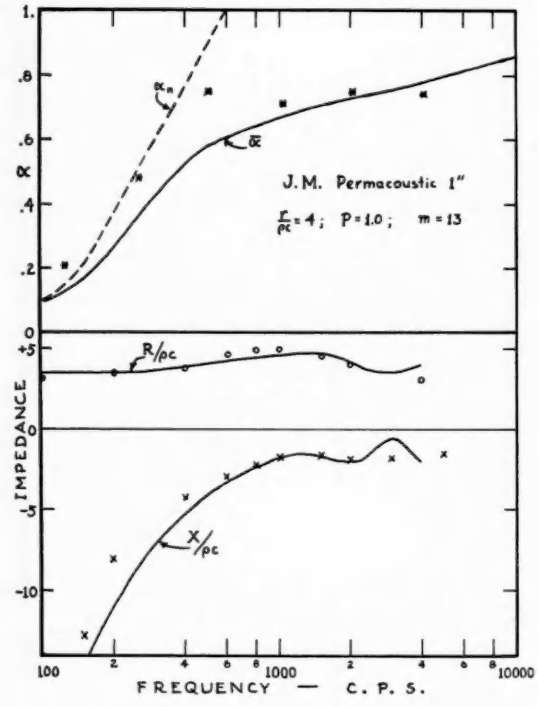

FIG. 1. Upper part: A.M.A. values of absorption coefficient compared with theoretical curves for normal and Sabine coefficient, obtained from experimental values of impedance. Lower part: Experimental values of impedance compared with theoretical curves for constant values of flow resistance, porosity and effective density given in figure.

practice, on the other. From the point of view of theory, both impedance and absorption coefficient are derived quantities, dependent on the physical properties of the absorbing surface. However, as we shall show in this paper, the train of logic leading from the flow resistance, porosity, etc., of the material to its absorption coefficient is so long a one as to make it practically impossible to determine these properties of the material by measuring the absorption coefficient. The acoustic impedance is, so to speak, placed midway between the physical properties and the absorption coefficient; as we shall show, it is possible to obtain values of effective porosity, etc., from the curves of impedance *vs.* frequency, and it is also possible to obtain reasonably accurate values of absorption coefficient from these same curves. Therefore, although the acoustical engineer will probably prefer to express the acoustical properties of a material in terms of its absorption coefficient, it is preferable for the acoustical scientist to express these properties in terms of the more fundamental acoustic impedance.

THE PHYSICAL PROPERTIES OF THE ABSORBING MATERIAL

It is possible to extend the ideas of Wintergerst, Gemant and others[2] to express the acoustic impedance of a porous material in terms of its physical properties.[3] Following these workers, we define the porosity P of the material to be the ratio of the volume of free air in the material to the total volume of the material. The effective resistivity or flow resistance r of the material can be defined as the ratio between the pressure drop per unit thickness and the volume flow of air through the material (in cc per sec. per cm² of surface). The quantity $(r/\rho c)$ has the dimensions

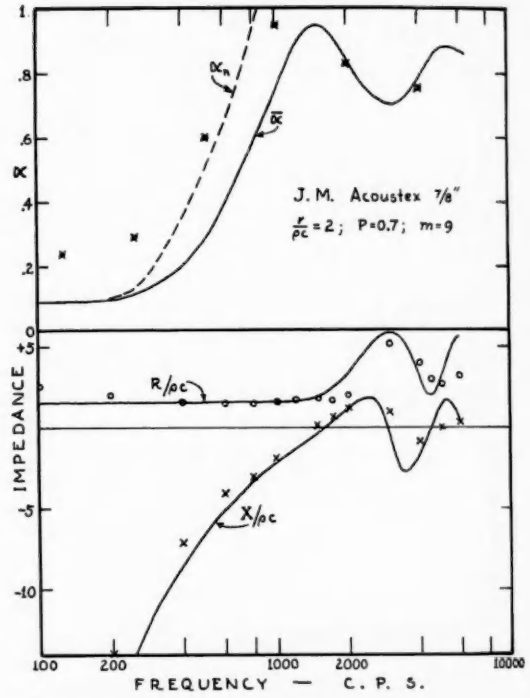

FIG. 2. Upper part: A.M.A. values of absorption coefficient compared with theoretical curves. Lower part: Experimental values of impedance compared with theoretical curves.

[2] E. Wintergerst, Schalltechnik **4**, 85 (1931). V. Kuhl and E. Meyer, Sitz. Preuss. Akad. wiss., Phys.-Math. Klasse 416 (1932). A. Gemant, Sitz. Preuss. Akad. wiss., Phys.-Math. Klasse 579 (1933). Rettinger, J. Acous. Soc. Am. **6**, 188 (1935).

[3] Details of the theoretical results discussed in this paper will be treated in an article by two of the writers (P.M.M., R.H.B.) to appear in Rev. Mod. Phys.

cm^{-1} and can be called the specific resistivity of the material. In addition one must define the effective mass of the air in the pores. The ratio of this mass to the mass of an equal volume of air in the open will be denoted by m and will be called the specific mass. As Kuhl and Meyer and Gemant[2] have pointed out, m is usually greater than unity, since part of the pore material moves with the air and adds its inertia. As a matter of fact none of these three quantities will have exactly the same values at acoustic frequencies as they have for steady state or geometrical measurements. Just as m is a constant giving the effective mass of the moving parts in the porous material, so P is a measure of the effective stiffness of the air and material combined. So defined, P is the ratio between the stiffness of air alone to the acoustic stiffness of the air and material combined and it is not surprising that such a quantity will change with frequency. The resistivity may also change with frequency, although it is possible that its value at acoustic frequencies may be nearly equal to the value measured for steady flow.

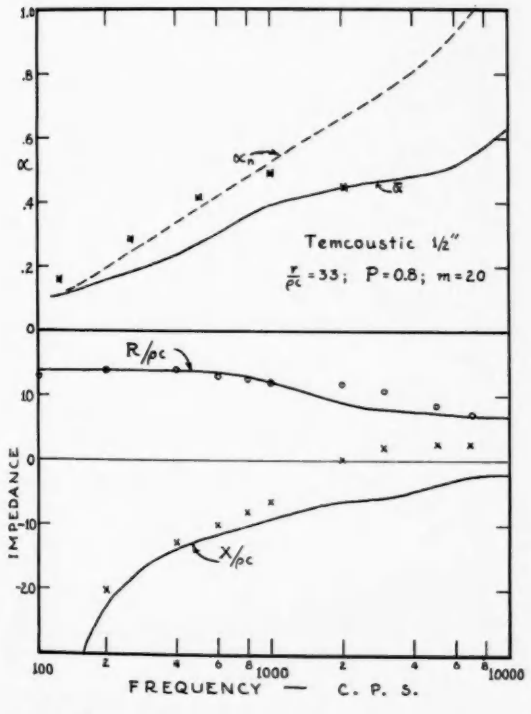

FIG. 3. Upper part: A.M.A. values of absorption coefficient compared with theoretical curves. Lower part: Experimental values of impedance compared with theoretical curves.

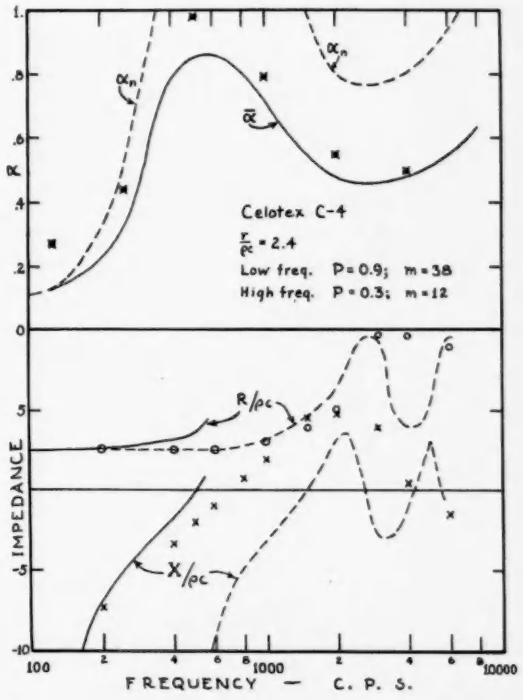

FIG. 4. Upper part: A.M.A. values of absorption coefficient compared with theoretical curves for normal and Sabine coefficient, obtained from experimental values of impedance. Lower part: Experimental values of impedance compared with theoretical curves for constant flow resistance, porosity and effective density. Solid curve for $P=0.9$, $m=38$; dashed curve for $P=0.3$, $m=12$.

If one assumes that the material is uniform and isotropic in porosity, etc., and that it has a rigid backing, the theory[3] indicates that the acoustic impedance of the material has the value Z given by the equation

$$2(Z/\rho c)(P/m)^{\frac{1}{2}}\cos\Phi_r = (a+ib)\tanh\pi[-i\sigma(a+ib)+\tfrac{1}{2}i], \quad (1)$$

$$a=\{2[1+(\gamma/\sigma)^2]^{\frac{1}{2}}+2\}^{\frac{1}{2}},$$

$$b=\{2[1+(\gamma/\sigma)^2]^{\frac{1}{2}}-2\}^{\frac{1}{2}},$$

$$\sigma=(L/\lambda)(mP)^{\frac{1}{2}}\cos\Phi_r,$$

$$\gamma=(rL/2\pi\rho c)(P/m)^{\frac{1}{2}}\cos\Phi_r.$$

Here L is the thickness of the material and λ the wave-length of the sound in free air. The angle Φ_r is the angle of refraction of the wave in the material, given in terms of the angle of incidence Φ_i by the equation

$$\cos\Phi_r=\{1-P^{-1}[m+i(r/\rho\omega)]^{-1}\sin^2\Phi_i\}^{\frac{1}{2}}.$$

Usually $\cos \Phi_r$ is close to unity for all values of Φ_i; so that Z is practically independent of the angle of incidence.

It is not expected that so simple a theory should give a detailed and accurate check with experiment. Most acoustic materials are not perfectly isotropic, and mounting conditions are difficult to reproduce. Moreover the variation of P and m with frequency would make it difficult to predict accurately the dependence of Z on ν. Consequently it is quite gratifying to find that more than half of Beranek's curves can be reproduced reasonably accurately by solutions of the above equation, using values of m, P and r *independent* of the frequency. Figs. 1 and 2 show typical cases. In the lower half of each figure the solid lines give the real and imaginary parts of the impedance computed from Eq. (1), using the values for the physical constants given in the figure. Crosses and circles give the experimental values obtained by Beranek.[1] The small discrepancies present may indicate a variation of P and m with ν, or they may be due entirely to the unavoidable inaccuracies in the measurements, method of mounting, etc.

For some materials it is obvious that their physical structure is complex enough that Eq. (1) will not adequately predict their impedance. For instance, some materials are less pervious at their surface than in the interior. Such structure would have the result, at lower frequencies, of adding a mass reactance in series with the term given by Eq. (1). This is presumably the case in Fig. 3, where the reactance as measured rises above the curve from Eq. (1) by an amount which increases with frequency. If this outer "skin" also has stiffness against bending, this would add, in series with the mass reactance, a stiffness reactance which increases with the cube of the frequency.[3]

The material Celotex C4, with its holes punched almost through the panel, and with the porosity in the hole lining considerably greater than that of the rest of the surface, presents a case which obviously cannot be represented by as simple a theory as that which gives Eq. (1). Since the conductivity of holes is greater the lower the frequency, presumably the effective porosity of the material should be that for the hole linings at low frequencies, and that for the front surface at high frequencies, having a value which decreases continuously as the frequency increases. The lower part of Fig. 4 has two sets of theoretical curves drawn, for two different (constant) values of the porosity. These indicate that the above considerations apply. Presumably a curve drawn for P and m changing continuously with frequency will fit the data reasonably well.

When the acoustic material is spaced out away from the rigid backing, leaving an air space behind it, an equation similar to Eq. (1) can be derived[3,4] which usually has a smaller stiffness reactance at low frequencies and which exhibits resonance peaks at lower frequencies than the curves for a rigid backing. In these cases also a mass and stiffness reactance must be added in parallel, due to the panel action of the slab of material. This effect is usually appreciable only at low frequencies. In addition the dependence of impedance on angle of incidence becomes no longer negligible.

In conclusion, we see that in the majority of cases an acoustical material has a constant resistance and a stiffness reactance (due to the stiffness of the air in the pores) inversely proportional to the porosity times the frequency at low frequencies. At higher frequencies resonance peaks occur which are more pronounced the smaller the value of the product $rL(P/m)^{\frac{1}{2}}$.

Sound Decay in Rooms of Simple Shape

It has been shown[5] that a knowledge of the acoustic impedance of the walls allows one to calculate the decay of sound in a simple rectangular room. Sound in such rooms can be considered to be made up of a number of standing waves, normal modes of air oscillation in the room. The amplitude of each wave depends on the source position and frequency, and the decay rate of each depends on its own distribution in space. Due to the regularity of the room shape, the smoothness of the walls, etc., a small number of these waves have a distribution such that the air motion is all parallel to two or four of the walls. These waves, which we can call the

[4] R. H. Bolt and R. L. Brown, J. Acous. Soc. Am. **12**, 31 (1940).

[5] P. M. Morse, J. Acous. Soc. Am. **11**, 56 (1939). F. V. Hunt, L. L. Beranek and D. Y. Maa, J. Acous. Soc. Am. **11**, 80 (1939). L. Cremer, Akustische Zeits. **5**, 46 (1940).

grazing modes, have smaller decay rates than the rest. Their decay rates can be calculated from the curves drawn in the previous papers[5, 6, 3] and an effective absorption coefficient α_0 can be computed, equal to twice the corresponding decay coefficient δ given in the curves. This coefficient is not the one defined by Sabine, and its value depends as much on the room shape as on the material of the walls. Nevertheless, its insertion in the Sabine formula enables one to compute the decay rate of these grazing modes. We can call α_0 the *grazing coefficient* for the material and room.

The previous papers have shown that in most cases all the waves which are not grazing one or more pairs of walls have the same decay rate, a rate corresponding to the coefficient for *normal incidence*. This coefficient, good for the majority of the waves, we can denote by α_n, and can call the *normal coefficient* of the material. It can be obtained from the charts in the previous papers, or from the simpler approximate formula

$$\alpha_n \backsimeq 8\rho cR/(R^2+X^2). \tag{2}$$

Where R is the real part of the acoustic impedance Z and X is the imaginary part. This approximate formula is accurate to three percent or less as long as the magnitude of $(Z/\rho c)$ is larger than two.

To the approximation inherent in Eq. (2) the normal coefficient of a material is independent of the room shape and size. In general its value is larger than the absorption coefficient used in practice, particularly at higher frequencies. The reason for this will be discussed later in this paper. This coefficient, likewise, is not the Sabine coefficient; in fact it is not strictly an *absorption* coefficient, for it may have values larger than unity. Therefore, we call it a normal coefficient and not a normal absorption coefficient.

In nearly all cases of practical interest, the normal coefficient is larger than the grazing coefficient. This means that the initial part of the decay curve[7] for sound in a regular rectangular room has a slope which corresponds to the normal coefficient, and represents the damping of the majority of the sound energy in the room. After a certain length of time this energy becomes negligibly small, leaving a residue of energy in the grazing modes, which dies out at a slower rate, corresponding to the grazing coefficient. The final part of the curve, therefore, is another straight line of smaller slope, which, when continued back, cuts the vertical axis ($t=0$) at a level corresponding to the ratio between the energy carried by the grazing modes and that carried by the other modes, expressed in terms of db.

These points are more clearly represented by an approximate formula, obtained from the previous papers[5, 6, 1] and from the work of Bolt and of Maa,[8] giving the decay of sound in a regular rectangular room with uniform coverage for each wall. Let the dimensions of the room be L_x, L_y, L_z, the areas of the walls be $S_x=L_yL_z$, etc., and the average grazing and normal coefficients for each wall pair be α_{x0}, α_{xn}, α_{y0}, etc., $(\alpha_{xn}=\frac{1}{2}\alpha_{xn1}+\frac{1}{2}\alpha_{xn2})$. Then the mean square pressure in the room (expressed in db) during the decay is the sum of three terms, D_x, D_y and D_z, $D=D_x+D_y+D_z$, where

$$D_x=10\log\left\{\frac{(2L_x/\lambda)-(\frac{1}{2})}{(\alpha_{xn}S_x+\alpha_{yn}S_y+\alpha_{zn}S_z)^2}\,\text{antilog}\,(-244\alpha_{xn}S_xt/V)\right.$$
$$\left.+\frac{\epsilon}{(\alpha_{x0}S_x+\alpha_{yn}S_y+\alpha_{zn}S_z)^2}\,\text{antilog}\,(-244\alpha_{x0}S_xt/V)\right\}$$
$$-10\log\left\{\frac{(2L_x/\lambda)-(\frac{1}{2})}{(\alpha_{xn}S_x+\alpha_{yn}S_y+\alpha_{zn}S_z)^2}+\frac{\epsilon}{(\alpha_{x0}S_x+\alpha_{yn}S_y+\alpha_{zn}S_z)^2}\right\}, \tag{3}$$

where the dimensions are in feet. This formula is valid for frequencies ν, band widths $d\nu$ and volumes V such that $(4\pi V\nu^2 d\nu/c^3)$ is larger than than fifty. The expressions for D_y and D_z can be obtained from Eq. (3) by cyclic interchange of

[6] P. M. Morse, J. Acous. Soc. Am. **11**, 205 (1939).

[7] By the term "decay curve" in this paper, we mean the curve of mean-squared pressure in db plotted against time in seconds.

[8] R. H. Bolt, J. Acous. Soc. Am. **10**, 228 (1939). R. H. Bolt, J. Acous. Soc. Am. **11**, 74 (1939). D. Y. Maa, J. Acous. Soc. Am. **10**, 235 (1939).

x, y and z in the subscripts. The quantity ϵ is equal to $(\frac{1}{4})$ when both source and microphone are at corners of the room, to $(\frac{1}{2})$ when one is at a corner and the other is moved around to obtain an average pressure amplitude, and is equal to unity if both source and microphone are moved about.

The form of D is to some measure determined by the sort of sound used to excite the room. For a pure tone, of course, not a large enough number of different standing waves will be excited to produce the average effect represented by Eq. (3). The usual warble tones and multitones[9] consist of a large number of tones with small frequency spacing between them, with almost random phase differences, and with amplitudes which are large within a certain frequency band and quite small outside the band. If the band is wide enough to satisfy the requirement given above, and if in addition the frequency spacing between the component frequencies in the warble tone is the same size or smaller than the average frequency difference between the natural frequencies of the room, then the formula for D is that given by Eq. (3). If, on the other hand, the spacing between frequencies in the warble tone is considerably larger than that between the natural frequencies of the room, then the denominators of each term in the equation for D should be $(\alpha_{xn}S_x+\alpha_{yn}S_y+\alpha_{zn}S_y)$, etc., to the first power, instead of to the square. Usually this produces a small difference only, since usually the numerators of the terms are the predominant factors.

The decay curve, to the approximation of Eq. (3), is thus a sum of three broken curves, each starting with the slope determined by a normal coefficient, and finishing with a slope determined by a grazing coefficient. A projection of the final straight line of each of the three component curves cuts the $t=0$ axis at a point determined by the ratio of the coefficients of the two antilog terms. The larger this ratio is, the farther will the initial straight line portion continue before the curve breaks to take on its final slope. If all walls have about equal absorption this ratio is approximately equal to $(2L/\lambda)-(\frac{1}{2})$, so that the lower the frequency (or the smaller the room) the shorter is the initial straight portion.

[9] W. L. Barrow, J. Acous. Soc. Am. **10**, 275 (1939).

In the case where one wall is very much more absorbent than the others, the D corresponding to this wall has a much greater slope than the other two, and its break will occur a much shorter time after $t=0$. In such cases it is usually impossible to continue measurement of pressure long enough to detect the breaks for the other two D's; so that the experimental curve exhibits the curved part for one D plus the initial straight line portions of the other two. In this case the formula discussed by Beranek[1] can be used. Experimental checks for Eq. (3) will be discussed later in this paper.

In some cases for high frequency or for a very soft wall surface not only the grazing waves, but also those standing waves which have an angle of incidence next nearest to 90° (corresponding to $n=1$ in the terminology of the earlier papers[5]) have a decay rate different from the rest of the normal modes. In this case another effective coefficient α_1, must be used, different from α_0 and α_n, and the expressions inside the curly brackets in Eq. (3) will have three terms each instead of two. The factor $(2L_x/\lambda)-(\frac{1}{2})$ in the first terms will become $(2L_x/\lambda)-(\frac{3}{2})$, and the extra terms for the two expressions will be

$$(\alpha_{x1}S_x+\alpha_{yn}S_y+\alpha_{zn}S_z)^{-2}\ \text{antilog}\ (-244\alpha_{x1}S_xt/V)$$

and

$$(\alpha_{x1}S_x+\alpha_{yn}S_y+\alpha_{zn}S_z)^{-2},$$

respectively. These unusual cases will be discussed in more detail elsewhere.[3]

Impedance and Absorption Coefficients

The previous discussion and the earlier papers[5] have shown that the reverberation in a perfectly regular rectangular room can be predicted once the acoustic impedance of the walls is known. This agreement between theory and experiment is gratifying, but it does not cast much light on the relation between acoustic impedance and the decay of sound in rooms of more irregular shape. The important question to be answered before impedance measurements become of practical use is the relation between the impedance and the absorption coefficient used in general practice. Unfortunately, this relation is not simple, and to clarify it we must study the methods of measurement of the usual absorption coefficient.

The original Sabine definition of absorption coefficient required a uniformly diffuse distribution of sound energy in the room for its validity, and postulated a strictly linear decay curve. It is often considered that the values of α given in the official bulletin of the Acoustical Materials Association correspond to this definition, though it seems unlikely that the correspondence is particularly good below 500 cycles per sec. Certainly, in a perfectly regular, rectangular chamber, sound is *not* diffusely distributed and the decay curve is often far from linear; therefore, we cannot expect the Sabine coefficient to equal either the normal or the grazing coefficient discussed earlier in this paper. It is, therefore, of interest to examine how the usual reverberation chamber is modified from a regularly shaped room in order to obtain a straight line decay curve whose slope gives the A.M.A. coefficients.

In usual practice scattering vanes and other obstacles are inserted in the reverberation chamber, which in some measure produce a more random distribution of sound than is possible in a room with regular shape. From the point of view of the wave theory of acoustics these scattering objects tend to perturb each normal mode of vibration, destroying their symmetry and tending to diffuse the sound more uniformly. A standing wave in such a room can be expressed in terms of a series of normal modes for a regularly shaped room, much as a general wave form can be expressed in terms of a Fourier series. Such series have been used by Maa[10] and by Feshbach and Clogston[11] in their studies of the perturbing effect of irregularities of distribution of absorbing material and of wall shape.

For small irregularities the series representing each normal mode has one large term and a number of smaller ones; in other words the perturbed standing wave is not much different from one of the waves for a room without irregularities. In this case each nongrazing wave will have a small component of grazing wave in it, so that its decay rate will be lowered, though only very slightly. Each grazing mode will have a number of components of nongrazing waves present, so that its decay rate will be increased. The increase will be somewhat greater than the decrease for the nongrazing modes, because usually there are many more nongrazing modes than there are grazing ones. The general decay curve will be more nearly a straight line.

However, when the scattering irregularities become large enough, the series for each standing wave will have a large number of terms of nearly equal size, and we can no longer say that one wave is a grazing one or not. In this case the decay rates for *all* the perturbed standing waves will be *nearly equal*, having a value intermediate between the grazing and the nongrazing decay rates for the simple room. When this is true the sound has become *truly diffuse*, the decay curve will be a straight line, and the Sabine formula will hold.

The theory of Feshbach and Clogston[11, 3] gives us some idea of the effectiveness of various wall irregularities in diffusing the sound. It indicates that inclining the walls at angles other than 90° has very little effect[12] as long as the walls remain smooth, and that irregularities of the order of a wave-length in size have an optimum effect. This has been mentioned by Meyer, and has been studied in detail for a special case by Pellam.[13] The theory also indicates that patches of absorbing material on the walls have a diffusing effect, though their effectiveness, as far as diffusing is concerned, is much smaller than that of "dents" or "bumps" of similar size in the wall. A symmetrical arrangement of irregularities in the room will not produce a straight line decay curve, for such arrangements cannot destroy symmetry in the normal modes, and waves of different symmetry will tend to have different decay rates. This has been pointed out by Maxfield and Potwin.[14]

One can see, therefore, that in a rectangular room with a relatively small amount of small scale irregularity the sound will be far from diffuse at low frequencies, whereas at high frequencies the irregularities become more effective and, eventually, adequate diffusion may result. In such a room the decay curve at low frequencies will be a broken line of the sort represented by

[10] D. Y. Maa, J. Acous. Soc. Am. **12**, 39 (1940).
[11] H. Feshback and A. M. Clogston, Phys. Rev. **57**, 1058A (1940).
[12] E. Meyer, J. Acous. Soc. Am. **8**, 155 (1937).
[13] J. R. Pellam, J. Acous. Soc. Am. **11**, 396 (1940).
[14] J. P. Maxfield and C. C. Potwin, J. Acous. Soc. Am., **11**, 48 (1939).

Eq. (3), and at high frequencies it will be a straight line corresponding to the Sabine formula. If the initial slope only of these curves were measured to obtain an absorption coefficient, the measured value would correspond to the normal coefficient at low frequencies, and would equal the true Sabine coefficient only at high frequencies, intermediate frequencies giving intermediate values.

Therefore it is apparent that one can define several kinds of absorption coefficients for a given material, each being related to a different decay rate by the Sabine formula, each rate manifesting itself under different experimental conditions. The grazing coefficient α_0 is not of general interest since its value depends as much on the room as on the material, and since its value can only be determined by measuring the lower reaches of the decay curve. The *normal coefficient* α_n, given by approximate Eq. (2), is usually independent of room shape or size, and is probably the quantity measured at low frequencies in reverberation chambers. There is also the classical absorption coefficient, given by the slope of the linear decay curve occurring in a room which is sufficiently irregular in shape to ensure random, diffuse sound distribution. In this case the theory[3,11] indicates that the definition of Sabine

$$\bar{\alpha}=2\int_0^{\pi/2}\alpha(\Phi)\cos\Phi\sin\Phi\,d\Phi \tag{4}$$

will hold.

Such a coefficient, for rooms with completely diffuse sound, we will call the *Sabine* or *statistical coefficient*, and denote it by the symbol $\bar{\alpha}$. Its connection with the impedance of the absorbing material is obtained by means of the familiar formula[15]

$$\alpha(\Phi)=1-\left|\frac{(Z/\rho c)\cos\Phi-1}{(Z/\rho c)\cos\Phi+1}\right|^2 =\frac{(4R/\rho c)\cos\Phi}{[(R/\rho c)\cos\Phi+1]^2+(X/\rho c)^2}. \tag{5}$$

When this is inserted in Eq. (4) the final formula for the Sabine coefficient becomes

$$\bar{\alpha}=\frac{8\rho cR}{R^2+X^2}\left\{1+\frac{\rho c(R^2-X^2)}{X(R^2+X^2)}\tan^{-1}\left(\frac{X}{R+\rho c}\right)-\frac{\rho cR}{R^2+X^2}\ln\left[\frac{(R+\rho c)^2+X^2}{(\rho c)^2}\right]\right\}, \tag{6}$$

where R and X are the real and imaginary parts of the acoustic impedance of the material, the inverse tangent is expressed in radians, and the symbol ln stands for the natural logarithm. This is probably the quantity measured in reverberation chambers at high frequencies, and probably the quantity measured in large irregular rooms at any frequency.

Finally there is the coefficient measured in reverberation chambers and reported in the bulletin of the Acoustic Materials Association. This we shall denote by $\alpha_{\text{A.M.A.}}$, and shall call the A.M.A. or *chamber coefficient*.[16] As the above discussion indicates, we should expect $\alpha_{\text{A.M.A.}}$ to approximate α_n at low frequencies and to approach $\bar{\alpha}$ at high frequencies.

To show that this is indeed the case, we have computed α_n and $\bar{\alpha}$ from Beranek's impedance curves for the four materials discussed in the first part of this paper. These quantities are plotted in the top parts of Figs. 1 to 4, together with the A.M.A. coefficients, marked as crosses, taken for mounting conditions as nearly similar to those used by Beranek as is possible. We see at once that $\alpha_{\text{A.M.A.}}$ is equal to $\bar{\alpha}$ for frequencies higher than 2000 c.p.s., within the errors inherent in the experiments and in the mounting conditions. In two of the cases the values of $\alpha_{\text{A.M.A.}}$ approach the curve for α_n at low frequencies, as was predicted above. In the other two cases the values of $\alpha_{\text{A.M.A.}}$ are higher than α_n. However, we must remember that in the reverberation chamber the material is mounted in slabs which are considerably larger in size than those used by Beranek in his measurements of impedance. If there is any energy lost by panel action, this will be most pronounced at the low frequencies, and a much greater loss will be exhibited by larger panels than by small ones. It is possible

[15] E. T. Paris, Proc. Phys. Soc., London, **115**, 407 (1927); Nature **9**, 126 (1930).

[16] F. V. Hunt, J. Acous. Soc. Am. **11**, 38 (1939). It would be more in the spirit of Professor Hunt's remarks to say that $\alpha_{\text{A.M.A.}}$ is one of the many possible chamber coefficients.

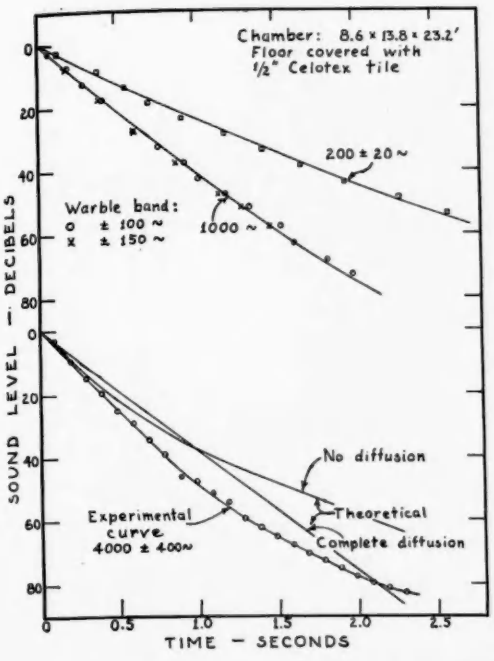

FIG. 5. Upper part: Experimental points for sound decay in rectangular chamber, compared with theoretical curves from Eq. (3). Lower part: Experimental decay curve compared with theoretical decay curve for no diffusion of sound and with Sabine curve for complete diffusion.

that this difference in size of samples, together with other differences in mounting conditions, will explain the whole of the difference between $\alpha_{A.M.A.}$ and α_n at low frequencies. The check for all the other materials measured by Beranek, for which values of $\alpha_{A.M.A.}$ (for nearly the same mounting conditions) are available, turns out to be as good as the four shown here. In every case $\alpha_{A.M.A.}$ is effectively equal to $\bar{\alpha}$ for frequencies above 2000 c.p.s. In half the cases $\alpha_{A.M.A.}$ is effectively equal to α_n for frequencies less than 500 c.p.s. and in the other cases $\alpha_{A.M.A.}$ is larger than α_n for very low frequencies, by an amount which can possibly be explained by differences in sample size and mounting conditions.

In the light of the foregoing discussion we can say that the absorption coefficient $\alpha_{A.M.A.}$ used in practice *does not* correspond to the coefficient defined by Sabine at low frequencies, though it does for frequencies above 2000 c.p.s. At very low frequencies the diffusing objects in the reverberation chamber are not sufficient to produce complete diffusion, and the decay curve is a broken line, although the first 30 db or so may be nearly straight. The slope of this initial straight portion, however, corresponds to the normal coefficient and not to $\bar{\alpha}$. For both the high frequencies and the very low, therefore, it is possible to compute $\alpha_{A.M.A.}$ from the values of the acoustic impedance. At 1000 c.p.s. $\alpha_{A.M.A.}$ lies between α_n and $\bar{\alpha}$, indicating considerable diffusion in the chamber but not enough to achieve completely random distribution. Here the relationship between $\alpha_{A.M.A.}$ and the acoustic impedance is very complicated indeed. It appears likely that absorption coefficients measured in rooms which are larger and more irregular than are the reverberation chambers would correspond to $\bar{\alpha}$ for lower frequencies than does $\alpha_{A.M.A.}$. This is partially corroborated by the observation[17] that field coefficients are usually smaller than those measured in reverberation chambers.

Reverberation Measurements in a Rectangular Chamber

In order to demonstrate the adequacy of Eq. (3) for the decay of sound in a rectangular chamber without diffusing obstacles, in order further to investigate the adequacy of Eq. (1) for the impedance of the simpler acoustic materials, and in order to detect the onset of diffusion at high frequencies, a series of decay curves has been made in the M.I.T. reverberation chamber[18] with ($\frac{1}{2}$") Celotex tile[19] covering the floor of the chamber. The curves were made with a Sound Apparatus Company high speed level recorder, using a beat frequency oscillator with warble condenser, a W.E. 555 speaker unit with 4-ft. exponential horn, and a Brush sound-cell microphone. Duplicate curves were traced[1] over each other on the tape, and to cover 80-db range curves were run with several amounts of attenuation in the microphone output. Each point in Fig. 5 is an average of 20 runs. Curves were first made for the empty chamber, and α_0 and α_n

[17] See the Symposium on Absorption Coefficients, J. Acous. Soc. Am. **11**, 37 (1939).

[18] We wish to thank the Department of Electrical Engineering at the Massachusetts Institute of Technology, and particularly Professors R. D. Fay and W. M. Hall, for aid in assembling the apparatus for these measurements.

[19] We wish to thank the Celotex Corporation for supplying the material used in these measurements, and, in particular, Dr. Hale J. Sabine for his help in this respect. Further work is being carried on with J.-M. Super Felt, supplied through the kind cooperation of Dr. J. S. Parkinson.

were determined for the bare walls by means of Eq. (3). The Celotex was then laid directly on the floor, so as completely to cover it, and curves were taken for 200, 500, 1000, 2000 and 4000 c.p.s.

The initial and final slopes of these curves were measured, and values of α_n and α_0 were computed. From these values curves for the D's were computed. These are compared with the data in Fig. 5, upper part, for 200 and 1000 c.p.s. The agreement between the theoretical curves and the experimental points provides a partial check of the adequacy of Eq. (3). Once the initial and final slopes are fixed, the shape of the curve for D, for a room of given size and for a given frequency, is completely fixed. The fact that the breaks in the experimental and the theoretical curves come the same distance along the curve shows that the factors multiplying the antilog terms in Eq. (3) are sufficiently accurate for our purposes. The curve for 500 c.p.s. checks with equal accuracy, and the correspondence for 2000 c.p.s. is nearly as satisfactory. The 4000 c.p.s. curves will be discussed later.

To complete the check with experiment, we must show that the values of α_n and α_0 exhibited by the experimental curves equal those computed from the impedance curve. In the present case, since experimental curves for impedance were not available, we have attempted a more long range check. From experimentally determined values of porosity, flow resistance and effective mass, we have computed a curve of impedance by means of Eq. (1), and from this theoretical curve we have computed values of α_n and α_0 to be compared with the experimental values.

By methods described by Gemant and Rettinger[2] we have measured the flow resistance of a sample of Celotex tile 64 sq. in. in area. This yielded a value of 1500 ± 300 c.g.s. units for the resistivity r of the material for steady flow. This, together with values of acoustic impedance measured at 200 and 500 c.p.s., serves to indicate that probable values of porosity and mass are $P=0.25$ and $m=250$. The probable error in these values is about 20 percent. Assuming that these three quantities remain constant for all frequencies, we have computed the impedance curves shown in the lower part of Fig. (6) from Eq. (1). From these predicted curves of impedance we have computed the curves of $\bar{\alpha}$, α_n and α_0 shown in the upper part of Fig. 6, using Eqs. (2) and (6) and the charts given in previous papers.[5,6] These curves are to be compared with the experimental points shown by circles and crosses. The circles are supposed to fit with the dashed line for α_n, and the crosses with the dot-dashed line for α_0.

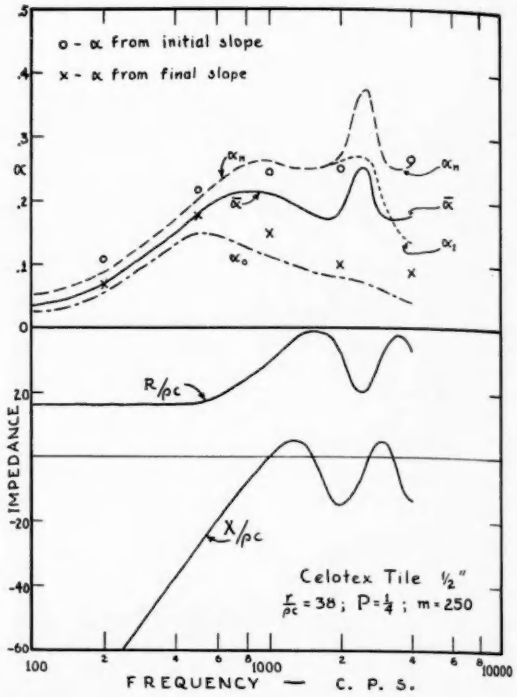

FIG. 6. Lower part: Theoretical curve for impedance for values of flow resistance, porosity and effective density given in figure. Upper part: Experimental values of α_n and α_0 obtained from initial and final slopes of decay curve, compared with theoretical curves obtained from theoretical impedance curve below.

The check is satisfactory, considering the large probable error of the assumed values of r, P and m. It is also possible that the dynamic value of r is not quite the same as the measured value for steady state flow, or that P and m are not independent of frequency, for this material. The measured values of α_n, taken from the initial slopes, follow the theoretical curve fairly well. The final slopes are all somewhat higher than the theoretical curve, but follow the general trend. The largest discrepancy is at 4000 c.p.s. which we must study in more detail.

The theory indicates that at 4000 c.p.s. there should be two sorts of wave with decay rates

differing from the normal rate; the grazing waves which move parallel to the absorbing surface, and the waves with the next largest angle of incidence. Corresponding to these are α_0 and α_1 on the plot. From the decay curves it is impossible to determine α_1, but the measured value of the final slope seems to give a value midway between the theoretical values for α_0 and α_1. To see whether Eq. (3) (or rather its generalization when three α's are different) is valid in this case, we have computed a decay curve using the theoretical values of the α's. This is shown in Fig. 5 at the bottom, as the curve marked "No Diffusion." It obviously does not fit the experimental curve, its break coming entirely too near the start of the curve. Even the modification mentioned in the discussion following Eq. (3), of using the first powers of the brackets in the denominators, will only lower the final part of the theoretical curve about 4 db, which is not enough.

We believe this discrepancy is an indication that the irregularities in the experimental chamber are sufficient to produce a noticeable diffusion of sound at this high frequency, though the diffusion is not enough for the Sabine reverberation to take place. The discussion earlier in this paper leads us to believe that as we increase the amount of diffusion in a room the decay curve will gradually change from the one represented by Eq. (3) to approach the straight line given by the Sabine formula. The first effect is to increase the *slope* of the final part, and to increase the *length* of the initial straight part of the curve. Later, as the scattering is increased, the less sensitive nongrazing modes will be altered, the initial slope will decrease, and the curve will finally become a straight line with slope given by $\bar{\alpha}$. The only scattering agents in the experimental chamber were the loudspeaker, the microphone, a door and window which project about 2 in. into the room, and irregularities in the laying of the 16-in. squares of Celotex tile on the floor. These were not enough to produce complete diffusion, but evidently were enough to effect materially the decay rates of the grazing modes, and to increase the length of the initial straight part of the curve. Measurements are being continued in order to study in more detail the change in the decay curves as scattering and diffusion are increased in the chamber.

It is obvious that a great deal more investigation will be required before the interconnection between acoustic impedance and the absorption coefficient used in practice can be said to be fully understood. The writers believe that, as this interconnection becomes more clear, the advantages, in utility and generality, of impedance curves for acoustic materials will also become more apparent.

Sound Absorption in Rectangular Ducts

LEO L. BERANEK
Cruft Laboratory, Harvard University, Cambridge, Massachusetts
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I. INTRODUCTION

IN a previous issue of the Journal, Mr. Hale J. Sabine[1] published data on the absorption of sound in various rectangular ducts at several frequencies. An approximate formula was presented by means of which the attenuation along the duct in decibels per foot may be calculated, namely,

$$\text{db/ft.} = 12.6\bar{\alpha}^{1.4}P/A, \qquad (1)$$

where $\bar{\alpha}$ is a "chamber" absorption coefficient[2] for the lining of a duct whose perimeter is P inches and cross-sectional area is A square inches. He gives data for six ducts having different P/A ratios,* each lined with a rigid rock wool sheet one inch thick. These data showed that the attenuation per unit length along the duct was approximately directly proportional to P/A in the frequency range of 256–1024 cycles for ducts whose ratio of transverse dimensions was not greater than 2 to 1. However, the values of the absorption coefficient $\bar{\alpha}$ which fit the data were often found to be somewhat different from those measured in a customary reverberation chamber. Also, at higher frequencies the attenuation per unit length did not follow in direct proportion to P/A.

In this paper it is proposed to apply to the data of Sabine a theoretical treatment of the attenuation of sound in ducts given by Professor P. M. Morse.[3] The theory will be extended to determine under what conditions the attenuation is in direct proportion to P/A. It will be seen that the newer concept of the specific normal acoustic impedance of the duct lining is a more satisfactory designation for the absorbing properties of the lining than the "chamber" absorption coefficient.

[1] Hale J. Sabine, J. Acous. Soc. Am. **12**, 53 (1940).
[2] F. V. Hunt, J. Acous. Soc. Am. **11**, 38 (1939).
* Six sizes and shapes were tested which included all combinations of 9, 12 and 18 inches.
[3] Philip M. Morse, J. Acous. Soc. Am. **11**, 205 (1939).

II. METHOD OF COMPUTATION

For those who have not read the mathematics in Morse's paper, it will be of help to outline the various steps necessary to calculate the attenuation of sound in a long rectangular duct with all walls similarly treated. Simply stated, one starts with a duct of given transverse dimensions uniformly lined with a material whose specific normal impedance is known. From these facts one calculates at each frequency two parameters μ and κ which in turn are related by a formula to an attenuation constant σ. Finally, σ is multiplied by $(54.6f/c)$ to obtain the attenuation in decibels per foot (or cm), where f is the driving frequency and c is the velocity of sound in feet (or cm) per second. For convenience in calculation, Morse has plotted two charts, one effecting the transformation from the impedance, frequency, and dimensions of the duct to the parameters μ and κ; the other the transformation from μ and κ to the constant σ. These are Figs. 2 and 1, respectively, in Morse's paper. Two cases are considered in detail here, namely, square and rectangular ducts.

Square ducts

The procedure in obtaining the attenuation of sound in db/ft. (or cm) in a square duct of cross section (l^2) having all walls similarly covered with a material whose specific normal impedance is $Z/\rho c = [(R/\rho c)^2 + (X/\rho c)^2]^{\frac{1}{2}} \cdot \exp[j \cdot \tan^{-1} X/R] = |Z|/\rho c \exp(j\Phi) = \gamma \exp(j\Phi)$ is as follows: (1) Divide Φ (in degrees) by 13.2; (2) multiply γ by 2 and divide by η, where $\eta = (2fl/c)$, f is the frequency and c is the velocity of sound in ft./sec. (or cm/sec.); (3) using these quantities, i.e., $(\Phi/13.2)$ and $(2\gamma/\eta)$ as abscissa and ordinate, respectively, in Morse's Fig. 2, obtain μ and κ; (4) multiply μ by 2.83 and divide by η and similarly for κ; (5) using these quantities, i.e., $(2.83\mu/\eta) = (\mu'/\eta)$ and $(2.83\kappa/\eta) = (\kappa'/\eta)$, in place of his μ/η and κ/η enter his Fig. 1 and read σ from the ordinate; (6) finally, use his formula (2) to determine the attenuation in db per foot

(or cm), namely,

$$\text{db/ft. (or db/cm)} = 54.6(\sigma f/c). \qquad (2)$$

The multiplication of γ by 2 and of κ/η and μ/η by 2.83 in place of using his γ, κ/η and μ/η is necessary because his charts are drawn for the case of a single absorbing wall, while here four wall coverage is considered.

Frequently it will be found for cases of small attenuation ($\sigma<0.1$) that Fig. 1 is not usable and that one must use approximate formulas to determine σ, namely,

$$\sigma \doteq (\mu'/\eta)\cdot(\kappa'/\eta)/\tau,$$
$$\tau \doteq [1+(\kappa'/\eta)^2-(\mu'/\eta)^2]^{\frac{1}{2}}.$$

Also it is often found the $\tau \doteq 1$, so that

$$\sigma \doteq (\mu'/\eta)\cdot(\kappa'/\eta). \qquad (3)$$

Rectangular ducts

For the case of a rectangular duct with all walls similarly absorbing, whose transverse dimensions are l_y and l_z, the procedure is more difficult and is as follows: (1) Divide Φ (in degrees) by 13.2; (2) compute $(2\gamma/\eta_y)$ and $(2\gamma/\eta_z)$, where $\eta_y=(2fl_y/c)$, etc.; (3) using these quantities obtain μ_y, κ_y, μ_z, κ_z from Morse's Fig. 2 as before; (4) multiply μ_y by 2. κ_y by 2, etc.; (5) using these quantities, i.e., $\kappa_y'=2\kappa_y$, $\mu_y'=2\mu_y$, etc., solve for σ from the two simultaneous equations,

$$\sigma^2-\tau^2=[(\mu_y'^2-\kappa_y'^2)/\eta_y^2]+[(\mu_z'^2-\kappa_z'^2)/\eta_z^2]-1,$$
$$\sigma\tau=(\mu_y'\kappa_y'/\eta_y^2)+(\mu_z'\kappa_z'/\eta_z^2),$$

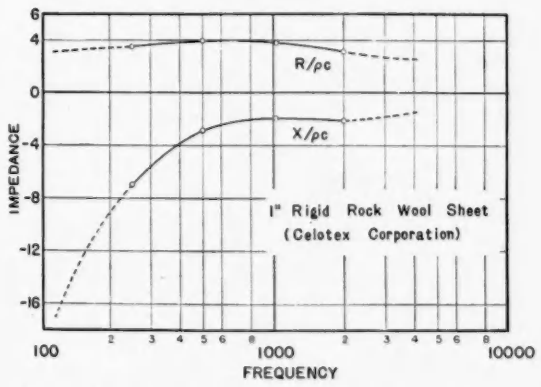

FIG. 1. Impedance curves computed from the data of Mr. Hale J. Sabine for a one-inch rigid rock wool sheet. The material was used to line rectangular ducts constructed of sheet metal panels.

no computational chart being available for these equations; (6) same as (6) above.

Large γ/η

For cases when $2\gamma/\eta>10$, Morse's Fig. 2 can no longer be used. Approximate formulas for calculating μ and κ are given in his Eqs. (8) except that attention should be called to an error existing in these as published. The first

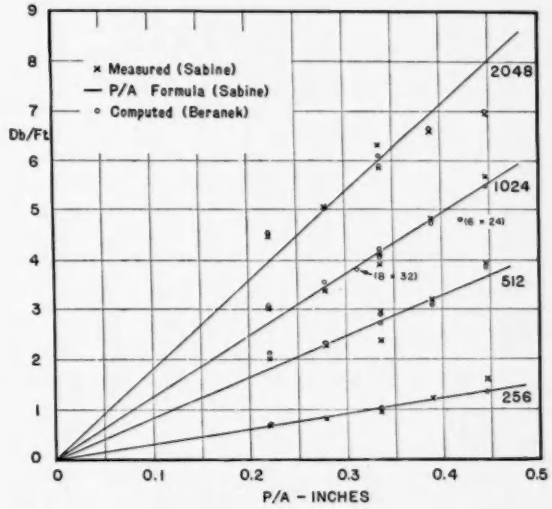

FIG. 2. Attenuation in db/ft. for six different sizes and shapes of ducts as shown in the following table:

DUCT	18×18	12×18	12×12 9×18	9×12	9×9
P/A	0.222	0.278	0.333	0.389	0.444

The upper circle at each frequency for $P/A=0.333$ is the computed attenuation for a 12×12-inch duct.

two of his Eqs. (8) correctly written are,

$$\mu \doteq \cos[45°-\Phi/2]\cdot[\eta/(\pi\gamma)]^{\frac{1}{2}},$$
$$\kappa \doteq \sin[45°-\Phi/2]\cdot[\eta/(\pi\gamma)]^{\frac{1}{2}}. \qquad (5)$$

In the case of a square duct, Eqs. (2), (3) and (5) can be combined, assuming $\tau \doteq 1$, to give

$$\text{db/ft.} = 17.4(\cos\Phi)/(\gamma l). \qquad (6)$$

Equation (6) is then similar to (1) above because P/A for square sections equals $4/l$. One is therefore justified in saying that *for small attenuations the P/A relation is valid for rectangular ducts not too far removed from being square. For all other cases accurate predetermination of the attenuation can only be achieved by using the exact theory.*

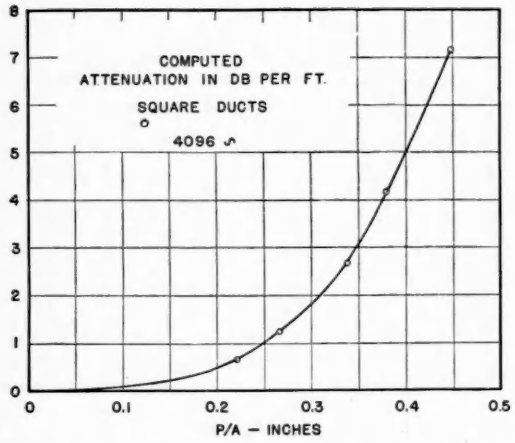

FIG. 3. Computed attenuations at 4096 cycles for square ducts 18, 15, 12, 10.6, and 9 inches on a side.

III. EXPERIMENTAL CONFIRMATION

Sabine's data at each frequency for two square ducts were used to determine γ and Φ of the absorbing walls by reversing the computational procedure outlined above. Then using these values of γ and Φ the attenuations were computed for the other four duct sizes investigated. A plot of $R/\rho c$ and $X/\rho c$ thus computed for the rigid rock wool sheet one-inch thick is shown in Fig. 1.[4] The computed attenuations for the ducts at several frequencies are shown in Figs. 2 and 3 by small circles; Sabine's data by crosses. It is seen that the exact theory fits the measured results better than the P/A formula.

The computations show that the attenuation is less for a rectangular duct of unequal transverse dimensions than for a square duct having the same P/A ratio. This is in agreement with earlier observations of Parkinson,[5] though Sabine apparently sometimes observes the reverse. Three cases are computed at 1024 cycles for rectangular ducts whose ratio of transverse dimensions is 1 : 4, namely, 8×32, 6×24, 4×16 inches. The attenuations for the first two appear in Fig. 2. For the 4×16 duct ($P/A=0.625$) the P/A formula predicts an attenuation of 7.7 db per foot, while the exact theory yields 5.8 db per foot. No experimental results are given to confirm these.

[4] Note how closely these curves resemble those for J.-M. Permacoustic presented in a previous paper, L. L. Beranek, J. Acous. Soc. Am. **12**, 14 (1940).

[5] John S. Parkinson, Johns-Manville Corp., Manville, N. J.

In Fig. 3 the computed attenuations for five square duct sizes at a frequency of 4096 are shown assuming the extrapolated values of $R/\rho c$ and $X/\rho c$ appearing in Fig. 1. The obvious deviation from the P/A law seen in this figure was observed experimentally by Sabine, though he did not publish his data. He says: "At 4096 cycles for the larger ducts the attenuation was much less than would be expected on the basis of direct proportionality to P/A."

Attention should also be called to the validity of the formula, given by Sabine,

$$\bar{\alpha}=1-(R-2\rho c)^2/(R+2\rho c)^2, \qquad (7)$$

which relates the chamber coefficient to the specific normal impedance R assuming it to be real. (7) is valid only for very small absorptions, say when $2\gamma/\eta>10$. This point was covered in a previous paper.[6] Also it is not correct to assume that the normal impedance is resistive. Table I, computed from Fig. 1, shows that the phase angle Φ is somewhat removed from zero.

At the lower frequencies the motion of the duct walls on which the lining is mounted contributes in large part to the attenuation of sound in the tube. This means that measurements on the normal impedance of the lining performed by a tube method with rigid wall backing cannot be used to predict accurately the attenuation experienced in practice. For this reason the extrapolated portions of the curves below 256 cycles in Fig. 1 lead one to predict smaller attenuations than were measured by Sabine. There is no way at present of estimating the magnitude of this type of attenuation except by setting up the full scale experiment.

IV. DUCTS WHOSE DIMENSIONS ARE LARGER THAN A HALF-WAVE-LENGTH

In addition to the traveling (or first type) wave existing in the duct, higher modes of vibration

TABLE I.

f	Φ^0	γ
256	−63.4	7.82
512	−36.6	4.86
1024	−27.7	4.30
2048	−35.6	3.80
4096	−31.0	2.91

[6] Hunt, Beranek and Maa, J. Acous. Soc. Am. **11**, 80 (1939).

will be excited when the wave-length becomes comparable to or less than twice the longest transverse dimension. Sound energy will be propagated along the duct by these "transverse" (higher types of) waves in addition to the energy carried by the first type of wave. However, the attenuation of the higher modes of vibration as a function of distance along the duct will in general be much greater than that of the first type, so that they are usually of little interest. A notable exception to this occurs when $2\gamma/\eta \doteq 1.18$ and $\Phi \doteq -40°$ where waves of the first and second type become identical, both having the same damping constant.* Calculation of the damping in decibels per unit length for the second type of vibration can be effected by entering Morse's Fig. 3 in place of his Fig. 2 as was described in Section II above. No charts are available for computing the attenuation of the third and higher types of waves.

Physically, when the damping constant for a "transverse" wave is not too much greater than that for the traveling or first type wave, one observes a double slope in the measured curve of sound pressure in decibels *vs.* distance from source end of the duct. The initial slope of the curve is associated with the damping constant of the higher mode of vibration, the final slope with that of the first type. A suggestion of a double slope is seen in the data of Sabine's Fig. 5 for a 9×12-inch duct at 2048 cycles.

* This phenomenon is discussed in greater detail by Professor Morse in the conclusion to his paper.

Transmission of Sound Through Parallel Conduits

L. W. LABAW
Brown University, Providence, Rhode Island
(Received July 15, 1940)

This paper is a theoretical study of the passage of sound through a series of similar sections composed of two parallel conduits. The infinite structure and the finite structure with finite termination are examined. The filter is low-pass and, by varying the cross sections and the lengths, practically as many or as few frequencies as desired, from 0 to 10,000 cycles, can be attenuated. The effect of cylindrical and conical finite terminations is calculated and plotted. The latter graphs indicate that by varying the length of the termination so that its anti-resonance frequencies fall within attenuation bands, the transmission is very approximately that predicted from the infinite structure study.

THE selectivity of two tubes in parallel for sound waves in air was first discussed by Herschel, and later employed by Quincke.[1] Lindsay and his collaborators[2] have treated the transmission of compressional waves in high-pass and low-pass filter structures. In the present paper the passage of plane harmonic compressional waves through a structure composed of recurrent similar sections, each composed of two parallel tubes, is examined. The first case considered is a structure with an infinite number of sections, and the second, a structure with a finite number of sections terminated by a finite tube of cylindrical or conical form.

I. INFINITE STRUCTURE

Consider an infinite number of sections, of the form represented schematically in Fig. 1, connected in series. Each section consists of two parallel tubes of lengths l_1 and l_2, respectively, and areas S_1 and S_2, respectively, inserted in a tube of uniform cross section S and length l at either end of the junction points. Quantities at the boundaries of the sections are denoted by the single subscripts 1, 2, 3, $\cdots$; quantities in the tubes of cross section S at the junction of three tubes, by the double subscripts 12, 21, 23, 32, $\cdots$; quantities in the tube of length l_2 at the junctions, by 122, 212, 232, 322, $\cdots$; and quantities in the tube of length l_1 at the junctions, by 121, 211, 231, 321, $\cdots$.

It is assumed that plane harmonic compressional waves traverse the structure and the resultant excess pressure and volume current at l are denoted by p_1 and $\dot{X}_1$, respectively. As usual the transmission is described by expressing the excess pressure and volume current at the end of any one section in terms of these quantities at the beginning of that section. In accordance with the common notation, ρ_0 denotes the mean density of the air, and $k=\omega/c=2\pi\nu/c$, where ν is the frequency and c the velocity of wave propagation.

If the acoustic resistance of a plane harmonic wave in the main line is denoted by $Z=c\rho_0/S$, the following relations are readily derived:[3]

$$\left.\begin{aligned} p_{12}&=p_1\cos kl-iZ\dot{X}_1\sin kl,\\ \dot{X}_{12}&=\dot{X}_1\cos kl-i\frac{p_1}{Z}\sin kl,\end{aligned}\right\}\quad(1)$$

$$\left.\begin{aligned} p_{211}&=p_{121}\cos kl_1-iZ_1\dot{X}_{121}\sin kl_1,\\ \dot{X}_{211}&=\dot{X}_{121}\cos kl_1-i\frac{p_{121}}{Z_1}\sin kl_1,\end{aligned}\right\}\quad(2)$$

$$\left.\begin{aligned} p_{212}&=p_{122}\cos kl_2-iZ_2\dot{X}_{122}\sin kl_2,\\ \dot{X}_{212}&=\dot{X}_{122}\cos kl_2-i\frac{p_{122}}{Z_2}\sin kl_2,\end{aligned}\right\}\quad(3)$$

$$\left.\begin{aligned} p_2&=p_{21}\cos kl-iZ\dot{X}_{21}\sin kl,\\ \dot{X}_2&=\dot{X}_{21}\cos kl-i\frac{p_{21}}{Z}\sin kl,\end{aligned}\right\}\quad(4)$$

[1] Lord Rayleigh, *Theory of Sound* (London, 1877–78), II, p. 58.
[2] R. B. Lindsay, "Filtration of sound," I, J. App. Phys. **9**, 615 (1938); II, *ibid.* **10**, 680 (1939); and references there cited.
[3] G. W. Stewart and R. B. Lindsay, *Acoustics* (New York, 1930), p. 335.

where the common time factor, $e^{i\omega t}$, has been omitted for convenience. The boundary conditions of continuity of volume current and pressure at each junction are

$$\left.\begin{aligned} p_{12}&=p_{121}=p_{122},\\ \dot{X}_{12}&=\dot{X}_{121}+\dot{X}_{122},\end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} p_{211}&=p_{212}=p_{21},\\ \dot{X}_{211}+\dot{X}_{212}&=\dot{X}_{21}.\end{aligned}\right\} \tag{6}$$

Equations (1) to (6) inclusive give fourteen boundary equations from which p_2 and $\dot{X}_2$ are to be expressed in terms of p_1 and $\dot{X}_1$. The result may be expressed in the form

$$\left.\begin{aligned} p_2&=Dp_1-iZE\dot{X}_1,\\ \dot{X}_2&=D\dot{X}_1-ip_1\frac{F}{Z},\end{aligned}\right\} \tag{7}$$

where

$$\left.\begin{aligned} D&=\left[A'\cos 2kl-\frac{B'Z^2+C'}{2Z}\sin 2kl\right],\\ E&=\left[A'\sin 2kl-ZB'\sin^2 kl+\frac{C'}{Z}\cos^2 kl\right],\\ F&=\left[A'\sin 2kl+ZB'\cos^2 kl-\frac{C'}{Z}\sin^2 kl\right]\end{aligned}\right\} \tag{8}$$

and

$$\left.\begin{aligned} A'&=\frac{Z_2\sin kl_2\cos kl_1+Z_1\sin kl_1\cos kl_2}{Z_2\sin kl_2+Z_1\sin kl_1},\\ B'&=\frac{2(1-\cos kl_1\cos kl_2)+\left(\frac{Z_1}{Z_2}+\frac{Z_2}{Z_1}\right)\sin kl_2\sin kl_1}{Z_2\sin kl_2+Z_1\sin kl_1},\\ C'&=\frac{Z_1Z_2\sin kl_1\sin kl_2}{Z_2\sin kl_2+Z_1\sin kl_1}.\end{aligned}\right\} \tag{9}$$

It can be shown that D, E, and F satisfy the relation

$$D^2+EF=1. \tag{10}$$

If we introduce the angle parameter W, such that $D=\cos W$, then $\sin W=(EF)^{\frac{1}{2}}$. The generalization of Eqs. (7) applied to any section becomes

$$\left.\begin{aligned} p_{n+1}&=p_n\cos W-iZ_0\dot{X}_n\sin W,\\ \dot{X}_{n+1}&=\dot{X}_n\cos W-i\frac{p_n}{Z_0}\sin W,\end{aligned}\right\} \tag{11}$$

where $Z_0=Z(E/F)^{\frac{1}{2}}$ is the usual characteristic impedance[4] of the line.

By the reasoning used in previous papers,[5] the filtration characteristics are given by the quantity

$$\cos W=\left[\left(\frac{Z_2\sin kl_2\cos kl_1+Z_1\sin kl_1\cos kl_2}{Z_2\sin kl_2+Z_1\sin kl_1}\right)\cos 2kl-\frac{Z}{2}\left(\frac{2(1-\cos kl_1\cos kl_2)+\left(\frac{Z_1}{Z_2}+\frac{Z_2}{Z_1}+\frac{Z_1Z_2}{Z^2}\right)\sin kl_2\sin kl_1}{Z_2\sin kl_2+Z_1\sin kl_1}\right)\sin 2kl\right]. \tag{12}$$

[4] Reference 2, I, p. 615.
[5] Reference 2, I, pp. 615–616.

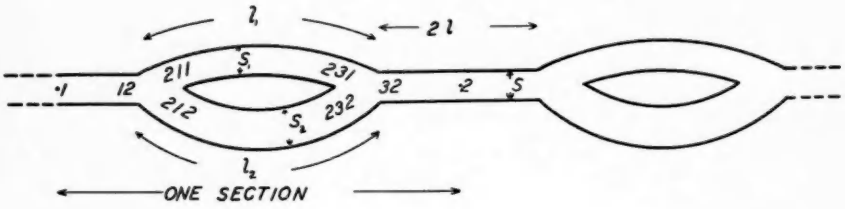

FIG. 1. Schematic representation of the filter structure.

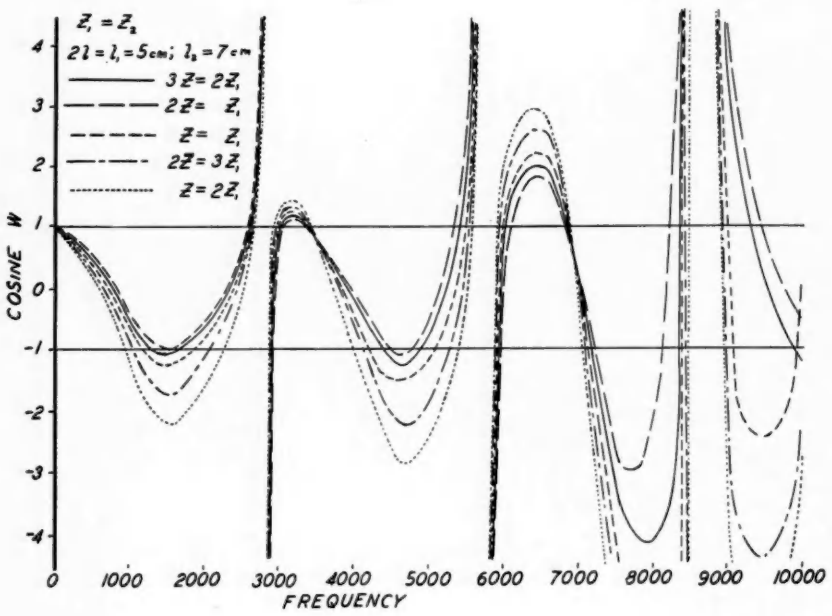

FIG. 2. Transmission characteristic curves for various ratios of the tube areas, keeping the lengths fixed.

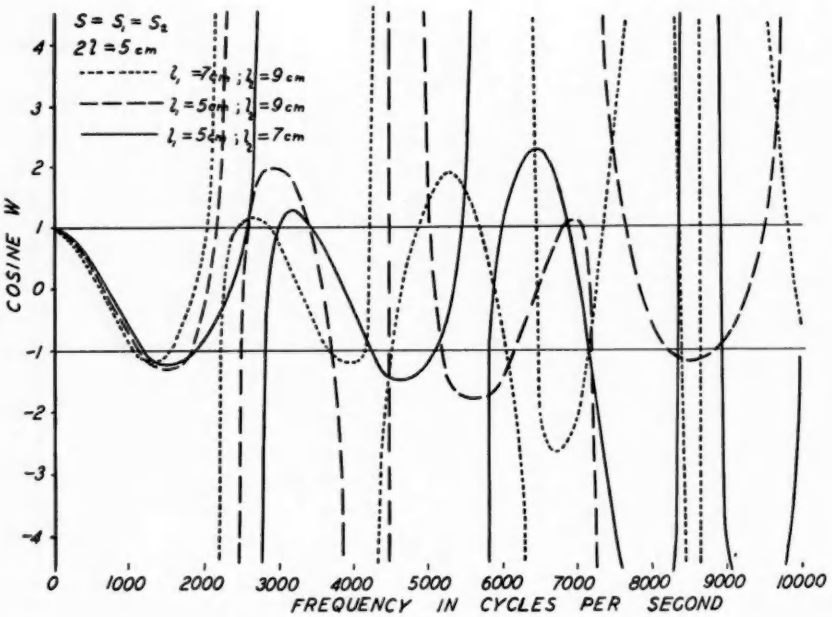

FIG. 3. Transmission characteristic curves for different tube lengths, keeping the areas fixed.

Transmission occurs for those frequencies for which $|\cos W| \leq 1$, while $|\cos W| > 1$ signifies attenuation. The evaluation of cos W for zero frequency indicates that the filter is low-pass. It is interesting to note that if either S_1 or S_2 is reduced to zero, the result is a special case of the stratified medium formula.[6]

Cos W is a function of all the areas of cross section and all the lengths. The dependence on the areas is shown in Fig. 2, where cos W is plotted as a function of frequency for $S_2 = S_1$, $l_1 = 2l = 5$ cm, $l_2 = 7$ cm, with the ratio S_1/S taking on the values $\frac{2}{3}$, $\frac{1}{2}$, 1, $\frac{3}{2}$, and 2. The curves indicate a minimum attenuation near $S_1/S = \frac{1}{2}$; from this value, the widths of the attenuation bands increase as the ratio becomes larger or smaller. The position of the infinite discontinuities is due to the lengths rather

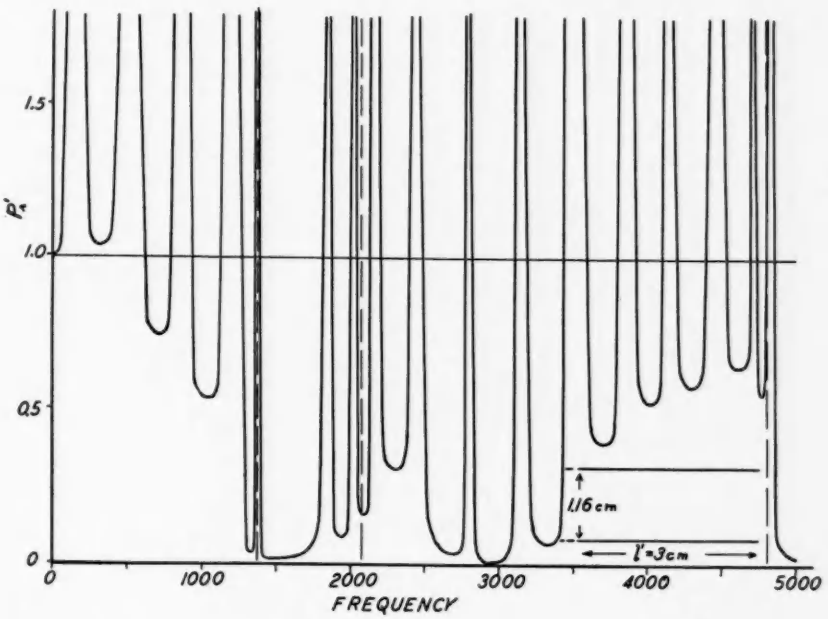

FIG. 4. Power transmission ratio for a finite filter terminated by an open cylindrical tube, neglecting the dissipation at the end.

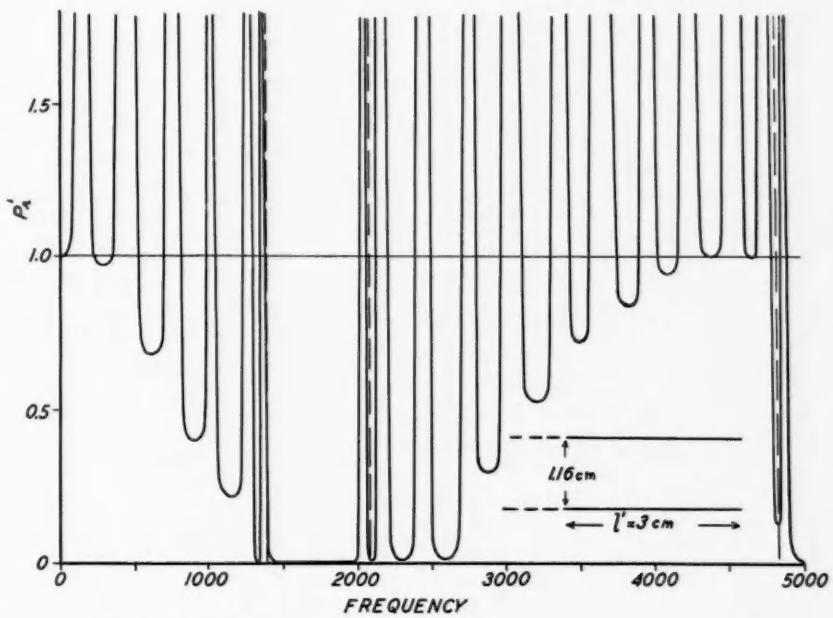

FIG. 5. The same as in Fig. 4, with dissipation at the end accounted for.

[6] R. B. Lindsay, C. R. Lewis and R. D. Albright, J. Acous. Soc. Am. **5**, 204 (1933).

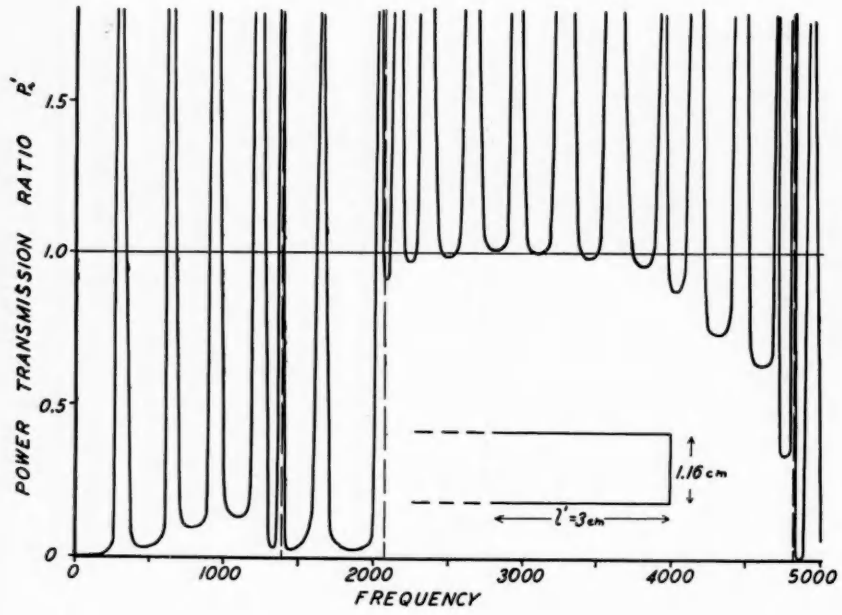

FIG. 6. Power transmission ratio for the finite filter of Fig. 4, terminated by a closed cylindrical tube.

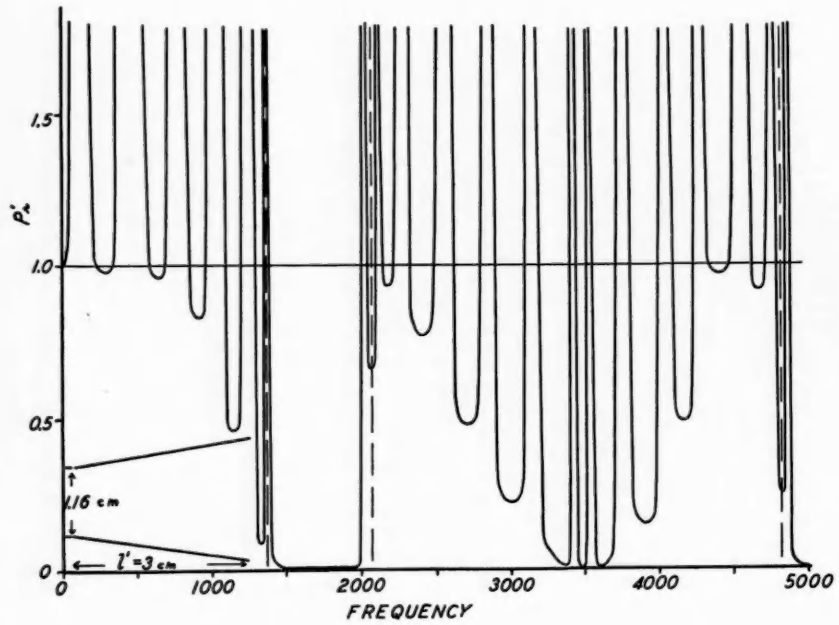

FIG. 7. Power transmission ratio for the finite filter of Fig. 4, terminated by an open frustum of a cone having the small end toward the filter, neglecting dissipation at the opening.

than the areas. The effect of changing the lengths is depicted graphically in Fig. 3, in which $S_1=S_2=S$, $2l=5$ cm for all the curves, and $l_1=5$ cm, $l_2=7$ cm, for the solid line. $l_1=5$ cm, $l_2=9$ cm, for the dashed line, and $l_1=7$ cm, $l_2=9$ cm, for the dotted line.

II. Finite Filters with Finite Endings

The theoretical calculations for finite filters by Lindsay[7] can be adapted to a finite number of sections, n, of the type shown in Fig. 1. Using for the power transmission ratio the form

$$P_r' = |\dot{\xi}_{n+1}|^2/|\dot{\xi}_1|^2, \tag{14}$$

[7] Reference 2, I, pp. 619–621.

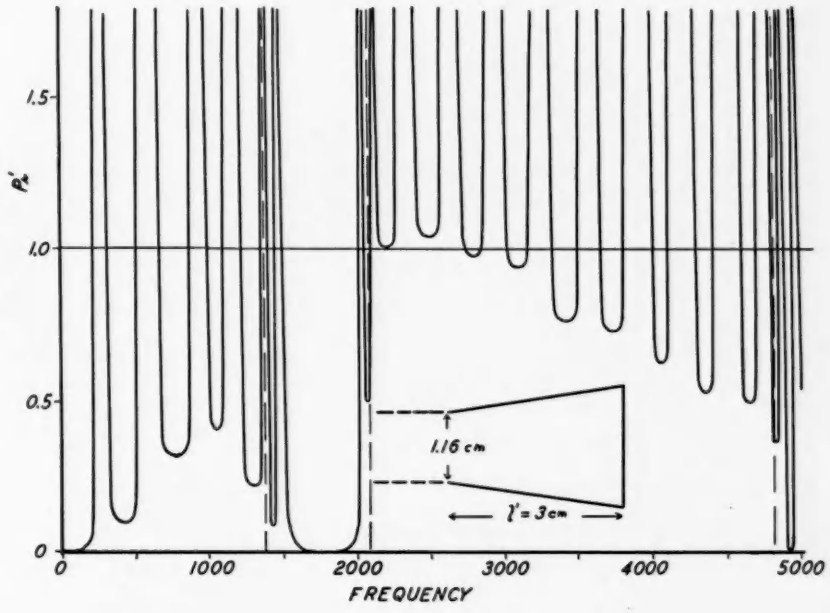

FIG. 8. Power transmission ratio for the finite filter of Fig. 4, terminated by a closed frustum of a cone with the small end attached to the filter.

where $\dot{\xi}_{n+1}$ and $\dot{\xi}_1$ are the particle velocities at the end of the last section of the finite filter and the beginning of the first section, respectively, we get equations similar to Eqs. (32) and (33) of Lindsay's paper.[7] Thus, for a termination in the form of a finite cylindrical tube with an open end, we get

$$P_r' = \left(\frac{S_i}{S'}\right)^2 \cdot \frac{1}{\left(\cos nW - \frac{Z'}{Z} F \tan kl' \frac{\sin nW}{\sin W}\right)^2}. \tag{15}$$

The primed quantities refer to the termination, and the subscript i to the line incident to the filter. The radiation resistance and the reactance of the open end are here neglected. If the open end is assumed to have an acoustical conductivity c_0 and radiation dissipation is also taken into account, the resulting ratio is

$$P_r' = (S_i/S')^2 \cdot (1/Q), \tag{16}$$

where

$$Q = \left\{ \cos nW - \frac{\left[\frac{\rho_0\omega}{c_0}(1-\tan^2 kl') - \left(\frac{\rho_0^2\omega^2}{c_0^2 Z'} - Z' + \frac{\rho_0^2\omega^2 k^2}{4\pi^2 Z'}\right)\tan kl'\right]}{R} \cdot \frac{F \sin nW}{Z \sin W} \right\}^2$$

$$+ \frac{\rho_0^2\omega^2 k^2 [1+\tan^2 kl']^2}{4\pi^2 R^2} \cdot \frac{F^2 \sin^2 nW}{Z^2 \sin^2 W}$$

and

$$R = \left[1 - \frac{2\rho_0\omega}{c_0 Z'} \tan kl' + \frac{\rho_0^2\omega^2}{(Z')^2}\left(\frac{1}{c_0^2} + \frac{k^2}{4\pi^2}\right)\tan^2 kl'\right].$$

We now note that P_r' does not become infinite as before though the maximum values are very large. An examination of Eq. (16) at the antiresonance frequency reveals that as a consequence of the termination the transmission does not drop to zero. A comparison of Figs. (4) and (5), in which Eqs. (15) and (16) are plotted, shows that the dissipation and reactance of the opening do not

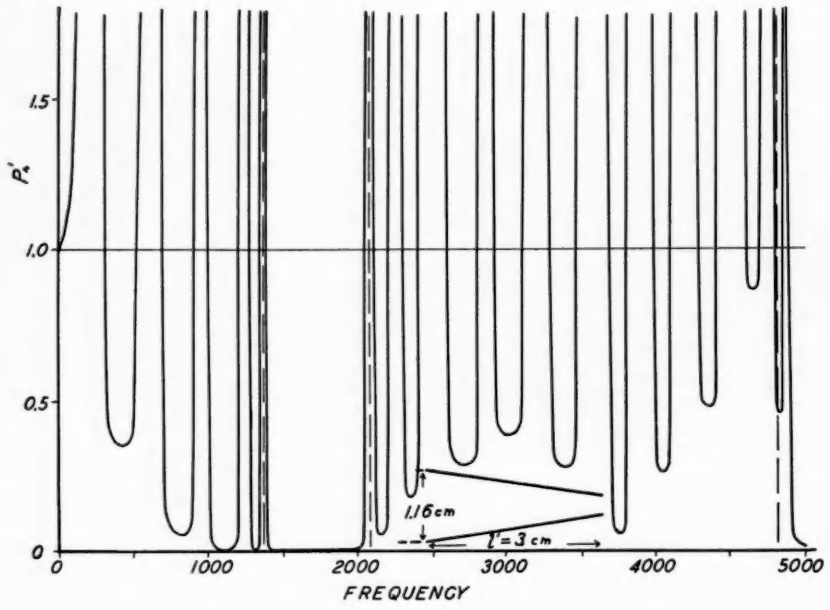

FIG. 9. Power transmission ratio for the finite filter of Fig. 4, terminated by an open frustum of a cone having the large end attached to the filter, neglecting dissipation at the opening.

appreciably increase the transmission at the antiresonance points, but do shift the antiresonance minima. In the figures shown, this effect is partly masked by the first attenuation band.

If the cylindrical tube termination is closed at the end, the transmission ratio is given by

$$P_r' = \left(\frac{S_i}{S'}\right)^2 \cdot \frac{1}{\left(\cos nW + \frac{Z'}{Z} F \cot kl' \frac{\sin nW}{\sin W}\right)^2}. \tag{17}$$

This is plotted as a function of frequency in Fig. 6. It will be noted that the line joining the minima varies as $\cos kl'$. In Fig. 4 to Fig. 11 inclusive, the dashed ordinates indicate the attenuation bands for the infinite filter. The values used in calculating the curves in Figs. 4, 5 and 6 are: $2l = l_1 = l_2 = 5$ cm; $n = 5$; $r = 0.58$ cm; $r_1 = 0.70$ cm; $r_2 = 0.40$ cm; $S_i = S' = S$; and $l' = 3$ cm (the symbol r refers to the radii of the tubes).

If the frustum of a cone is used as a finite termination, either the small or the large end may be attached to the filter. The frustum has an axial length l', an impedance at the end away from the filter $Z_{l'}$; and slope m. The subscript 1 refers to the small end of the frustum, and the subscript 2 to the large end. From geometry, $l' = (x_2 - x_1)$, $x_1 = S_1^{\frac{1}{2}}/m\pi^{\frac{1}{2}}$ and $x_2 = S_2^{\frac{1}{2}}/m\pi^{\frac{1}{2}}$; x_1 and x_2 being the respective distances from the small and large ends of the frustum from the vertex of the right circular cone from which the frustum is cut. The problem is to express Z_t, the impedance at the end of the last section of the filter, in terms of $Z_{l'}$. This calculation is carried out in Chapter Six of Stewart and Lindsay's *Acoustics*.[8] In using the result, their notation and definition of symbols has been adopted.

The ratio of the particle velocities is,[9] if S'' is the area of the termination next to the filter,

$$\frac{\dot{\xi}_{n+1}}{\dot{\xi}_1} = \frac{S_i}{S''} \cdot \frac{1}{\cos nW + i\frac{Z_t}{Z} F \frac{\sin nW}{\sin W}}. \tag{18}$$

[8] Reference 3, Chapter VI.
[9] Reference 2, II, p. 620.

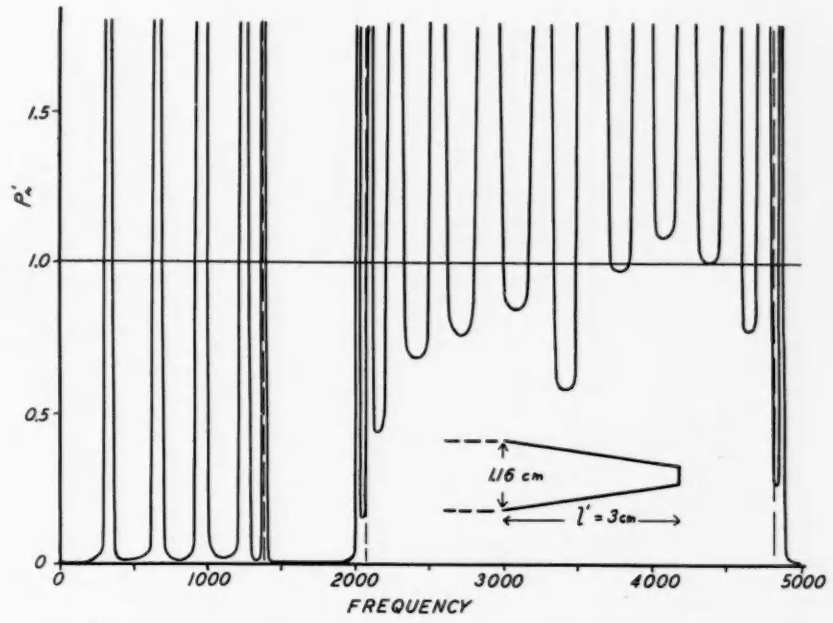

FIG. 10. Power transmission ratio for the finite filter of Fig. 4, terminated by a closed frustum of a cone with large end toward the filter.

For an open terminating frustum with small end to the filter, $Z_t = ib/a\omega$ (neglecting dissipation as is done in all the cases following) and the transmission ratio (Eq. (14)) reduces to

$$P_r' = \left(\frac{S_i}{S_1}\right)^2 \cdot \frac{1}{\left[\cos nW - \frac{k^2\rho_0 c^2 x_1}{\omega S_1(1+kx_1 \cot kl')} \cdot \frac{F \sin nW}{Z \sin W}\right]^2}. \tag{19}$$

The graphical representation of this is reproduced in Fig. 7, for values given below.

The closed terminating frustum having the small end to the filter gives $Z_t = ig/f\omega$. The ratio, which is plotted in Fig. 8 is

$$P_r' = \left(\frac{S_i}{S_1}\right)^2 \cdot \frac{1}{\left\{\cos nW - \frac{k^2\rho_0 c^2}{\omega S_1} \cdot \frac{x_1[1-kx_2 \cot kl']}{[1+k^2x_1x_2-kl' \cot kl']} \cdot \frac{F \sin nW}{Z \sin W}\right\}^2}. \tag{20}$$

If the frustum has the large end to the filter, and the small end open, the power transmission becomes, with $Z_t = -ib/g\omega$,

$$P_r' = \left(\frac{S_i}{S_2}\right)^2 \cdot \frac{1}{\left[\cos nW - \frac{k^2\rho_0 c^2 x_2}{\omega S_2(1-kx_2 \cot kl')} \cdot \frac{F \sin nW}{Z \sin W}\right]^2}. \tag{21}$$

The results of calculating Eq. (21) are shown in Fig. 9.

When a closed frustum presents the large end to the filter, $Z_t = -ia/f\omega$. The transmission for this case is calculated as

$$P_r' = \left(\frac{S_i}{S_2}\right)^2 \cdot \frac{1}{\left\{\cos nW - \frac{k^2\rho_0 c^2}{\omega S_2} \cdot \frac{x_2(1+kx_1 \cot kl')}{[1+x_2x_1-kl' \cot kl']} \cdot \frac{F \sin nW}{Z \sin W}\right\}^2}. \tag{22}$$

Figure 10 is the graph of Eq. (22).

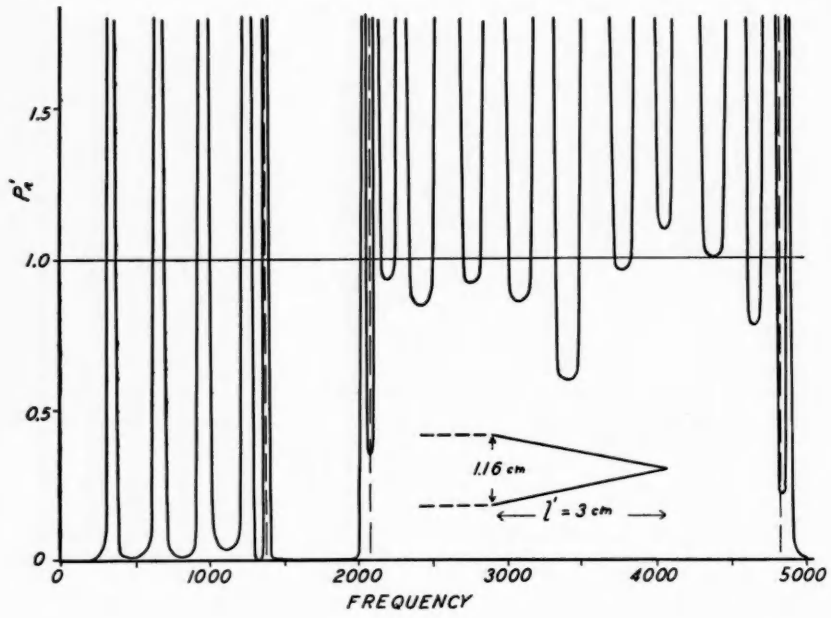

FIG. 11. Power transmission ratio for the finite filter of Fig. 4, terminated by a cone with the base attached to the filter.

The final case is the finite filter terminated by a closed cone. For this, $x_1=0$, and $Z_t=-ia/f\omega$ again. The power transmission ratio simplifies to

$$P_r'=\left(\frac{S_i}{S_2}\right)^2\cdot\frac{1}{\left\{\cos nW-\dfrac{k^2\rho_0c^2x_2}{\omega S_2[1-kl'\cot kl']}\cdot\dfrac{F\sin nW}{Z\sin W}\right\}^2}. \tag{23}$$

A comparison of Eq. (23) plotted in Fig. 11 with the other endings indicates that the closed cone is better than the others studied, except in the first transmission region.

The values used in calculating the curves for Fig. 7 to Fig. 11 inclusive are the same as used in Fig. 4, 5 and 6; in addition, the slope of the cone for Fig. 7 to Fig. 10 was 0.145, while for Fig. 11, it was 0.193. By adjusting the length of the ending to make the antiresonance points come in an attenuation region, good transmission could be obtained for any particular region.

The author is indebted to Professor R. B. Lindsay for the general filtration theory and for valuable suggestions during the completion of these calculations.

The Absorption of Sound in Carbon Dioxide

ROBERT W. LEONARD
University of California, Los Angeles, California
(Received September 3, 1940)

The absorption was measured by a direct method over frequencies between 22 and 112 kc. A microphone responding to the sound pressure is moved away from a piston source located in a flat surface. The output of the microphone is amplified and recorded photographically. The resulting pressure-distance curve yields the pressure attenuation coefficient. The measurements were made in carbon dioxide which was carefully dried by passing it through phosphorus pentoxide. The results of the measurements bear out the theoretical work of Bourgin. The lowest value obtained for the frequency at which the absorption per wave-length μ is a maximum was 30 kc. In order to account for the maximal value of μ obtained experimentally, it is necessary to assume that both the symmetrical valence vibration and the deformation vibration are effective in producing the absorption, and in addition that the second harmonic of the deformation mode also participates in the absorptive process. Velocity measurements made with the same apparatus have shown a reasonable agreement between the dispersion and absorption in carbon dioxide.

THE direct method of measuring the absorption of sound in gases has been used by both Grossmann[1] and Schmidtmüller.[2] The method is essentially that in which a microphone is moved away from the source, and the sound pressure is recorded for various distances. By a comparison of the actual pressure degradation with that to be expected for a dissipationless medium, the pressure attenuation coefficient for the medium is obtained. With a piston source in a large close-fitting plane baffle, the pressure attenuation coefficient β is given by

$$-\beta=\frac{d}{dx}\log\frac{p}{\sin\frac{k}{2}((x^2+a^2)^{\frac{1}{2}}-x)},$$

where p is the measured pressure amplitude at an axial distance x from a piston of radius a and k is equal to $2\pi/\lambda$ where λ is the wave-length in the medium. β can be obtained most conveniently from the slope of a logarithmic plot of

$$p\Big/\left[\sin\frac{k}{2}((x^2+a^2)^{\frac{1}{2}}-x)\right].$$

A set of these linear curves is to be seen in Fig. 1.

The apparatus used by the author is quite similar to that used by Schmidtmüller[2] although somewhat smaller. The essential difference lies in the treatment of the walls to reduce the reflected sound. Glass baffles are used instead of felt. Fig. 2 shows schematically the arrangement of the apparatus. The measuring chamber is cylindrical and has a diameter of 30.5 cm and a length of 66 cm. Measurements are made over a range of approximately 15 cm with a minimal distance from the source of about 6 cm. The sound pressure is measured by a Rochelle salt microphone about 1 cm square. The voltage produced is amplified and recorded on photographic paper. The microphone is raised and lowered by a magnetic drive which eliminates leakage and reduces the volume change to zero.

Before entering the evacuated measuring chamber, the gas passes through 4 meters of

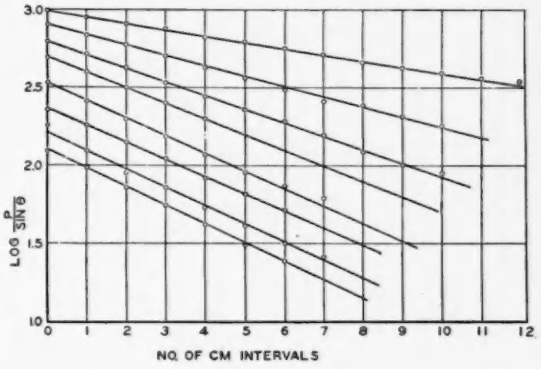

FIG. 1. Linear logarithmic curves from which m, the absorption per cm, is obtained.

[1] E. Grossmann, Ann. d. Physik **13**, 681 (1932).
[2] N. Schmidtmüller, Akustische Zeits. **3**, 115 (1938).

50-mm tubing containing P_2O_5 in a state of vigorous agitation. The entire set of measurements at the nine frequencies available were made without changing the sample. The gas received no further treatment besides this drying. The gas on which the measurements were made was obtained from the Pure Carbonic Corporation and was specified to be 99.5 percent pure prior to filling. The impurities were supposed to be due to air in the tank. This carbon dioxide is produced by the combustion of natural gas, and it is conceivable that it might contain traces of hydrogen which would not be removed in the drying process.

Figure 3 shows typical examples of the two types of curves obtained. The upper two are absorption curves and show the manner in which the sound pressure depends on distance. The lower pair shows the type of curve from which

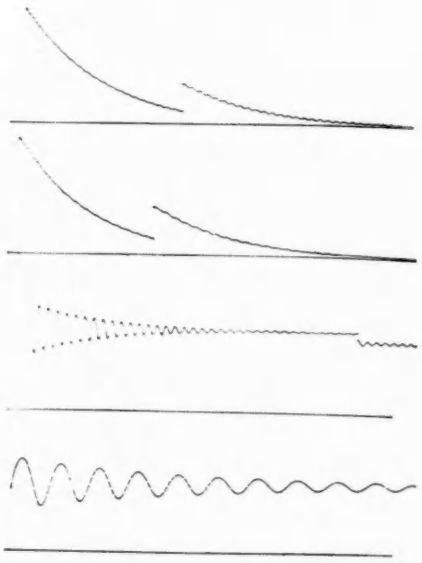

FIG. 3. Typical curves recorded for absorption and dispersion measurements.

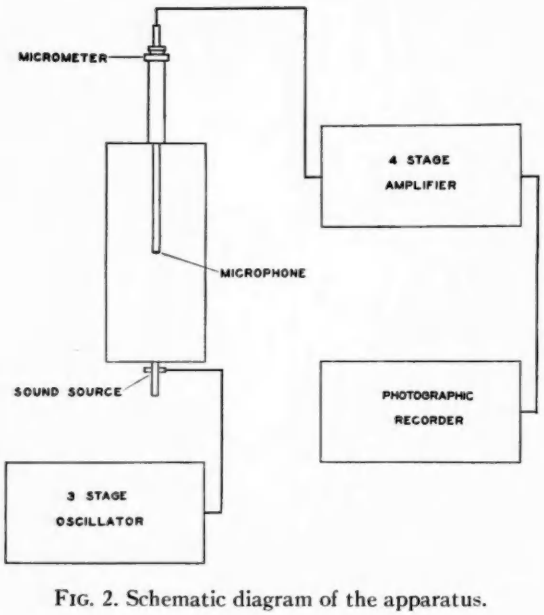

FIG. 2. Schematic diagram of the apparatus.

wave-length measurements were taken. This type of curve is produced by introducing a voltage directly from the oscillator into the microphone amplifier. The peaks are one wave-length apart. A reasonably accurate determination of velocity was possible using the wave-length determined from this type of curve and the source frequency determined by a comparison with the carrier frequency of a radio station. The dispersion curve from a set of measurements made on a single gas sample is shown in Fig. 4.

The absorption coefficient in reciprocal centimeters m which is equal to 2β was determined from logarithmic plots of

$$p \Big/ \left[\sin \frac{k}{2}((x^2+a^2)^{\frac{1}{2}}-x)\right]$$

as mentioned earlier. μ the absorption per wave-length was taken as the product of the experimental values of m and λ. A plot of μ against log frequency is shown in Fig. 5. This curve represents a set of data taken on a single gas sample. The smooth curve through the data represents the expected theoretical curve. It is a plot of Bourgin's Eq. (9.4)[3] with the constants adjusted to give a reasonably good fit. The measurements were made at or corrected to a temperature of 25° centigrade and a pressure of 763 mm of mercury. The value of μ_{max} from this curve is 0.251. Three other similar sets of data yield values of 0.255, 0.246, and 0.253, giving an average value of 0.251. The peak frequencies show much more uncertainty varying from the lowest at 30 kc to the highest at 45 kc. This variation indicates the presence of small varying amounts of impurities or incomplete drying and

[3] D. G. Bourgin, Phys. Rev. **50**, 355 (1936).

point to the possibility of a peak position below 30 kc.[4]

Bourgin's Eq. (9.4) may be written in the following form using the notation of Richards[5]

$$\mu=\frac{2\pi RC_i}{C_\infty(R+C_\infty)}\cdot\frac{\omega\theta}{\omega^2\theta^2+1+\dfrac{C_i}{C_\infty}+\dfrac{C_i}{R+C_\infty}+\dfrac{C_i^2}{C_\infty(R+C_\infty)}},$$

where the quantities are defined as follows: $\mu=m\lambda$=absorption per wave-length, R=gas

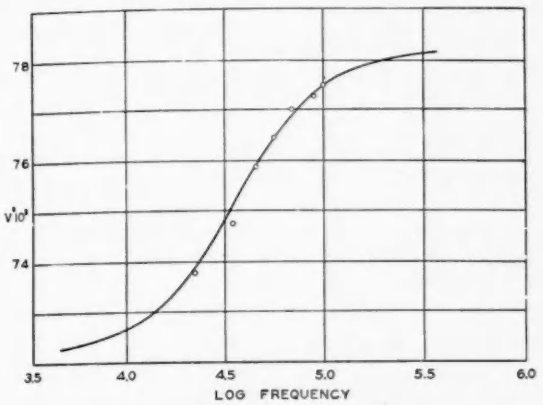

FIG. 4. Velocity squared against log frequency. The smooth curve represents the theoretical curve for $C_i=1.85$.

constant in calories per mole, C_i=the part of the specific heat at constant volume leaving the acoustical cycle, $C_\infty=C_v-C_i$, $C_v=C_p-1.027R$, $C_p=8.889$ at 25°C, $\omega=2\pi f$ when f is the sound frequency, and θ=average lifetime of a quantum of vibrational energy associated with the mode or modes of vibration leaving the acoustical cycle to produce the absorption.

The numerical value of C_v was determined from a value of C_p computed by Kassel[6] from a theoretical consideration of an elastic rotator. With C_v determined, a value can be obtained for C_i from the foregoing equation using 0.251 as the value for μ_{max}; C_i is readily obtained by successive approximations. The value obtained is 1.85 calories per mole.

[4] E. Fricke, working with V. O. Knudsen, has found that the peak position may be as low as 20 kc. See following article in this journal.

[5] W. T. Richards, Rev. Mod. Phys. **11**, 36 (1939).

[6] L. S. Kassel, J. Am. Chem. Soc. **56** (1939).

In order to compare the above value with what is to be expected on a theoretical basis, an examination of the energy level diagram for the molecule is necessary. The case of CO_2 is complicated by the fact that there are three modes of vibration: the δ or deformation mode which can occur in two planes, the symmetrical linear or valence vibration, and the unsymmetrical valence vibration. The contribution to the vibrational specific heat of the unsymmetrical linear vibration is beyond detection with the present technique. However, the contributions of lower levels of the deformation and symmetrical linear mode fall well within the limits of experimental error.

In computing the vibrational specific heats due to the deformation and symmetrical linear modes, the Fermi degeneracy between the second level of the δ mode and the first vibrational level of the s mode must be considered. The perturbation between these two levels gives rise to a splitting and mixing of their identities. This matter has been treated by Dennison[7] who has been kind enough to furnish me with the weights corresponding to each level. Table I gives the levels with their energies expressed in wave numbers and the weights as well as the fractional identity of each.

Taking these levels, one can write out a partition function and include as many terms as the available spectroscopic data will permit. However, it is not necessary to include many to account for a value for C_i of 1.85. For the purpose of computation, the levels of mixed identity may be separated by taking half of the total weight

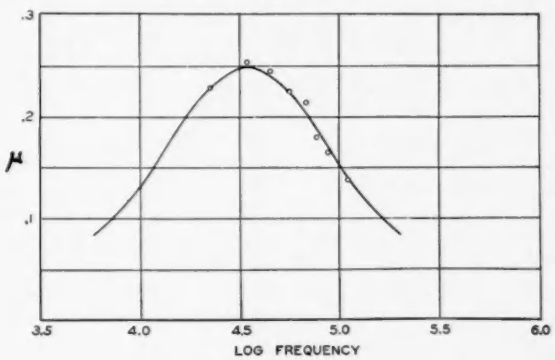

FIG. 5. A plot of μ the absorption per wave-length showing the symmetrical bell-shaped curve predicted by theory.

[7] D. D. Dennison, Phys. Rev. **41**, 310 (1932).

TABLE I. *Vibrational energy levels.*

ν_n IN CM^{-1}	$\theta_n=\frac{hc\nu_n}{k}$	MODE OF VIBRATION	TOTAL WEIGHT
667.5	955.3	δ	2
1285.8	1840	$\frac{1}{2}\delta$, $\frac{1}{2}s$	1
1336.2	1912	δ	2
1388.4	1987	$\frac{1}{2}\delta$, $\frac{1}{2}s$	1
1933.5	2770	$\frac{1}{2}\delta$, $\frac{1}{2}s$	2
2004.6	2860	δ	2
2077.1	2980	$\frac{1}{2}\delta$, $\frac{1}{2}s$	2

assigned to the mixed level. While such a procedure may seem questionable, it has been carried out to prove that the δ mode alone cannot account for the experimental value of C_i. The partition function, utilizing actual energy levels, has the form

$$Q=\sum_{n=0}^{n=m} g_n e^{-\theta_n\sigma},$$

where g_n is the weight assigned to the nth level, θ_n is the characteristic temperature of the nth level ($\theta_n=h\nu_n/k$) and σ is the reciprocal of the absolute temperature. The specific heat is given by

$$C=R\sigma^2\frac{d^2}{d\sigma^2}\log Q=R\sigma^2\left[\frac{1}{Q}\frac{d^2Q}{d\sigma^2}-\frac{1}{Q^2}\left(\frac{dQ}{d\sigma}\right)^2\right].$$

The partition function for the δ mode alone has the following form

$$Q=1+2e^{-\theta_1\sigma}+\tfrac{1}{2}e^{-\theta_2\sigma}+2e^{-\theta_3\sigma}+\tfrac{1}{2}e^{-\theta_4\sigma}+e^{-\theta_5\sigma}+2e^{-\theta_6\sigma}+e^{-\theta_7\sigma}$$

and a computation of C_i including these 7 levels gives a value of 1.79 calories per mole which is over 3 percent lower than the experimental value of 1.85. A partition function including only first and second levels of the δ mode and the first pair of mixed levels has the form

$$Q=2e^{-\theta_1\sigma}+e^{-\theta_2\sigma}+2e^{-\theta_3\sigma}+e^{-\theta_4\sigma}.$$

A calculation of C_i from this function, gives a value of 1.86 calories per mole which is in much better agreement with the experimental value than the contribution of the δ mode alone.

CONCLUSION

From the data presented in this paper, it is probable that the frequency at which the absorption per wave-length is a maximum is at least as low as 30 kc. In addition, it is highly probable that the average lifetimes of vibrational quanta associated with either the deformation mode or the symmetrical linear mode are identical or nearly so. This seems also to be true for the second level of the deformation mode, in that the contributions of both modes up to the second group of the system of levels must be included to account for the observed vibrational specific heat. That the two modes of vibration do not have separate and distinct lifetimes is not difficult to believe in view of the strong perturbing effect of one mode on the other and the consequent mixing of the identies of the resulting doublet levels. This interpretation from absorption data is in agreement with that of Eucken and Becker[8] based on dispersion measurements.

The dispersion data included in this paper, although less reliable than those from absorption measurements, are in reasonable agreement with the absorption data, at least so far as concerns the number of vibrational modes required to account for the observed dispersion.

Acknowledgment is due to Professor V. O. Knudsen for his guidance during the research, to Mr. L. P. Delsasso and Mr. Henry Jung for their help in the design and construction of certain parts of the apparatus, and to various members of the staff of the Department of Physics at the University of California at Los Angeles for their suggestions and cooperation.

[8] A. Eucken and R. Becker, Zeits. f. physik. Chemie **B27**, 219 (1934).

The Absorption of Sound in Five Triatomic Gases*

EDWIN F. FRICKE
University of California at Los Angeles, Los Angeles, California
(Received September 3, 1940)

The sound absorption coefficients have been measured between 8 and 130 kilocycles for the five triatomic gases CO_2, N_2O, COS, CS_2, and SO_2. The frequencies of maximal absorption for these molecules were found to be at 20, 153, 287, 379, and 1040 kc, respectively. It was found that for the linear molecules CO_2, N_2O, COS, and CS_2 a linear relationship exists between the maximal absorption coefficients and the frequencies at which these maxima occur. It was also found (1) that the lower the fundamental frequencies of vibration of the four linear molecules the higher are the acoustical frequencies of maximal absorption, i.e., the shorter are the lifetimes of the energy quanta; and (2) that the sonically activated fundamentals and harmonics of each of the above linear molecules commute and have the same lifetime, with the possible exception of CO_2. From the experimental data it was possible to calculate the reaction rates, probabilities of removal of vibrational quanta, the numbers of collisions necessary to remove the quanta of energy, and the numbers of quantum transitions per second. In addition to the above data, a technique is presented which enables one to find the frequencies of maximal absorption for gases when these frequencies occur in a range beyond the scope of the apparatus.

THE APPARATUS

THE apparatus used in these studies was of the same general design as that described in a previous paper.[1] However, the large sound chamber previously used was limited to use at frequencies lower than those employed in the present investigation. Also, the large chamber presented difficulties in the purification of the large volumes of gases needed, and in the inadequacy of sound diffusion even for moderately low frequencies. In addition, a new oscillator was required to extend the measurements to frequencies as high as 130 kc. The new oscillator is based on the recent Pierce magnetostriction generator.[2] See Fig. 1.

With proper adjustments of the grid and plate coils and tuning condenser, nickel rods, one-half inch in diameter, and of the required lengths to give various fundamentals and harmonics, could be made to oscillate with large amplitudes for frequencies as high as 130 kc.

The sound was admitted to the chamber through an air-tight glass tube that fitted over the nickel rods and extended from their midsections to the end plate of the sound chamber. The sound chamber is shown in Fig. 2. It was made from a brass cylinder 30 cm long and 21 cm inside diameter, with walls and end plates $\frac{1}{4}$ inch thick. Around the outside of the cylinder was mounted a heater compartment which could be filled with oil or glycerine, and maintained at a constant temperature by means of a heating coil.

The fan F served to diffuse the sound and also mix the gases in the chamber. It was found necessary to make its blades as irregular as possible; it then became effective in preventing undesirable shifts in the sound pattern in the chamber when the density of the gas was changed. The fan rotated at the rate of one revolution per second.

The microphone M was mounted upon the face of a hollow brass sphere 3.8 cm in diameter. The sphere, in turn, was mounted upon the end of a pendulum 10 cm long and swung by an electrically driven crank D through an arc of 60°, at the rate of one oscillation per second.

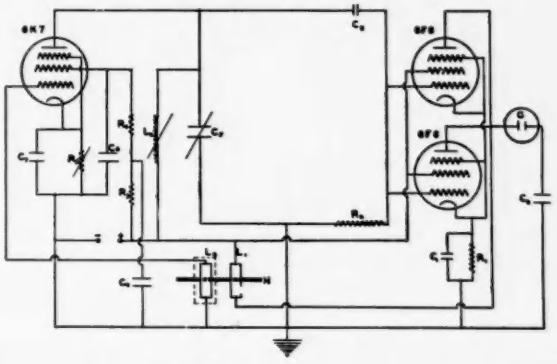

FIG. 1. Schematic design of supersonic oscillator.

* Doctorate Dissertation, University of California at Los Angeles, 1940.

[1] V. O. Knudsen and E. F. Fricke, J. Acous. Soc. Am. 10, 89 (1938).

[2] G. W. Pierce and A. Noyes, J. Acous. Soc. Am. 9, 185 (1938).

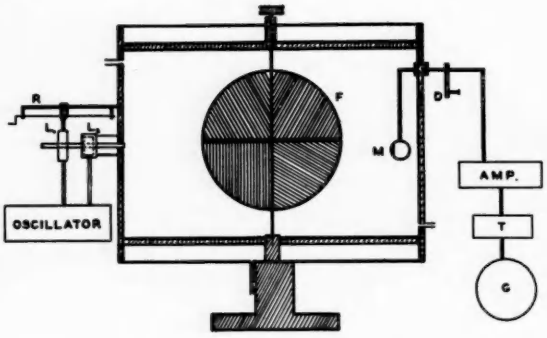

FIG. 2. Arrangement of apparatus.

The microphone itself was a Rochelle salt sound cell of the Brush type.

The microphone and associated four-stage amplifier, thermocouple T, and galvanometer G were calibrated at all the frequencies used. The calibration curves thus obtained made it possible to determine the pressure of the sound upon the crystal microphone in terms of the deflection of the galvanometer.

TECHNIQUE OF MEASUREMENT

The equations for the experimental determination of the absorption coefficient per cm, m, have been developed in a previous paper.[1] The general absorption equation can be presented as

$$m_x=(P_n/P_x)^2(\rho_x c_x/\rho_n c_n)^2 \times[(1-J_x)/(1-J_n)(\alpha S/4V+m_n)]-\alpha S/4V,$$

where α is the absorption coefficient of the walls of the chamber; S is the total surface area of the chamber; V is the total volume; P_n/P_x is the ratio of the sound pressure on the microphone when the chamber is filled with pure nitrogen to the sound pressure for a gas mixture; c_x and c_n are the velocities of sound in the mixture and in pure nitrogen; ρ_x and ρ_n are the corresponding densities of the gas mixture and pure nitrogen; J is a Bessel function; m_n is the absorption per centimeter in pure nitrogen, depending upon the frequency used, and is assumed to obey classical absorption.

The ratio P_n/P_x was determined from the deflections of the galvanometer, which indicated the pressure on the microphone, first when the chamber was filled with pure N_2 (which gives P_n) and then again when the chamber was filled with the gas mixture (which gives P_x). P_n and P_x, thus obtained, especially at low frequencies, must be appropriately corrected owing to the reflection and diffraction from the microphone and its mountings. The ratio $(\rho_x c_x/\rho_n c_n)$ was corrected for dispersion whenever the gas x was dispersive. The value of α was determined from an investigation of the effects of impurities upon the absorption of sound in oxygen, since the values of m for oxygen are well known. Hence Eq. (1) can be solved for $\alpha S/4V$ directly. α was found to be 0.023 for brass at 24 kc.

The percent of gas added to the nitrogen in the sound chamber was ordinarily obtained by displacing a definite volume of the absorptive gas into the chamber and then applying a correction for the resulting fractionation of the gas mixture. In the case of vapors like carbon disulphide, known weights were vaporized inside the chamber.

THE EXPERIMENTAL DETERMINATION OF THE ABSORPTION COEFFICIENTS FOR CARBON DIOXIDE

The carbon dioxide gas used was obtained in the commercial form and was purified by slowly passing it into the purification chambers shown in Fig. 3. The chamber A contained $Pb(NO_3)_2$ solution, chamber B contained $NaHCO_3$ solution, flask C is an empty chamber, D contained highly concentrated H_2SO_4, and flask E served as a displacement basin into which H_2SO_4 was allowed to flow from burette H. The gas was dried by passing it through $CaCl_2$ contained in T and then over P_2O_5 contained in G, J, and R. The entire purification chamber was pumped out for eight hours by pump P. The gas was then allowed to flow through the entire chamber from A to R to insure the washing out of any impurities that the pump failed to remove. When the chambers were completely filled, the stopcocks at O and R were closed and the gas was then allowed to stand in the drying chambers for two days. It was necessary to shake the P_2O_5 chambers at periodic intervals to prevent hard surfaces from forming over the powder and thus render it ineffective as a drying agent. The manometer F served to indicate atmospheric pressure inside the apparatus.

The nitrogen, which was used as the basic or standard gas, because it has only classical absorption at room temperature, was purified by passing it through concentrated H_2SO_4 and then over a course of 20 feet of P_2O_5 contained in glass tubes, $1\frac{1}{2}$ in. in diameter.

In the process of obtaining data on the absorption coefficient of CO_2 the sound chamber was first filled with pure N_2, and then known amounts of CO_2 were added to the sound chamber. The observed ratios of sound pressures P_x/P_n were obtained for various percentages of CO_2 and at a large number of frequencies of sound between 8 and 130 kilocycles. The observed ratios are shown in Fig. 4.

When the reciprocals of these ratios were introduced in Eq. (1) the values of m_x for the various percentages of CO_2 in N_2 and at different frequencies are obtained. These values are shown in Fig. 5. Owing to the linear graphs the absorption coefficients for 100 percent CO_2 can be obtained from the extrapolated straight lines. For frequencies as high as 100 kc one can obtain reliable values of P_x/P_n with as much as 14 percent CO_2 in the sound chamber. At frequencies below 24 kc the sound is sufficiently diffuse even for 100 percent CO_2. When the extrapolated values for 100 percent CO_2 are multiplied by the appropriate wave-lengths the values of μ, the absorption coefficients per wave-length, are obtained. When these values of μ are plotted against the logarithms of the frequency the bell-shaped curve shown in Fig. 6 is obtained. This graph shows that the absorption is a maximum at about 20 kc. The successive measurements of μ at each frequency fluctuated slightly, owing, no doubt, to unavoidable impurities. This is to be expected when it is found[3] that only one percent of water vapor will shift the position of the absorption peak 2250 kc. The magnitude of these fluctuations is indicated by the spread of the experimental points in Fig. 6.

The theoretical curve for the absorption per wave-length can be obtained by use of the Bourgin equation (or Kneser's) where

$$\mu = 2\pi \frac{RC_i}{[C(C+R)C_\infty(C_\infty+R)]^{\frac{1}{2}}} \frac{\nu_m \nu}{\nu_m^2+\nu^2} \quad (2)$$

R is the gas constant; C is the specific heat at constant volume, which is 6.8856 cal./mole at 23°C; C_∞ is the specific heat at constant volume that is independent of the sound frequency, which is 5.1461 cal./mole; C_i is the sonically activated vibrational specific heat, which according to the present results, includes only the deformation vibration and its second harmonic and has the value of 1.7395 cal./mole (the spectroscopic data were obtained from Dennison's work);[4] ν is the acoustical frequency; ν_m is the frequency of maximal absorption.

The magnitude of the maximal absorption is obtained from the equation

$$\mu_{\max} = \pi RC_i/[C(C+R)C_\infty(C_\infty+R)]^{\frac{1}{2}}, \quad (3)$$

which gives 0.230, which is 8.4 percent lower than the value obtained by Leonard.* The average lifetime of the energy quanta is obtained from the equation

$$\beta = (2\pi\nu_m)^{-1}[C(C+R)/C_\infty(C_\infty+R)]^{\frac{1}{2}}, \quad (4)$$

which is 10.26×10^{-6} sec.

Measurements on the Absorption of Sound in COS

Carbonyl sulphide was prepared by dropping sulphuric acid (5 parts of acid to 4 of water) on

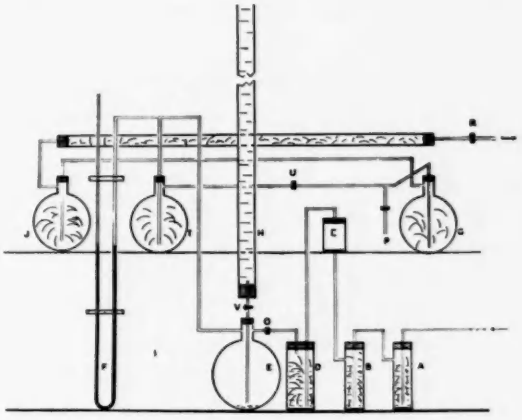

Fig 3. Apparatus for purifying gases.

[3] V. O. Knudsen and E. F. Fricke, "The Absorption of sound in CO_2, N_2O, COS, and in CS_2, containing added impurities." See following article in this journal.

[4] D. Dennison, Rev. Mod. Phys. **3**, 380 (1931); Phys. Rev. **38**, 2077 (1931).

* Preceding article, this issue.

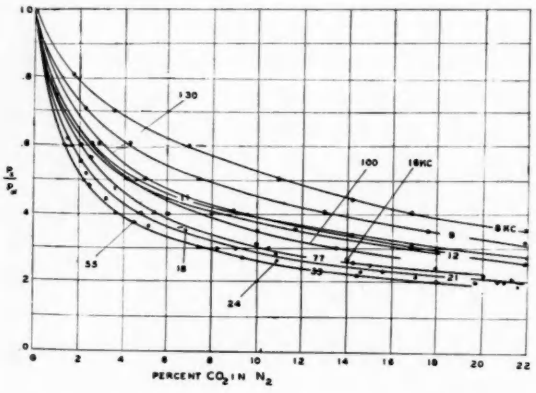

FIG. 4. Curves showing the relative pressures (as measured by the microphone) for CO_2 in N_2.

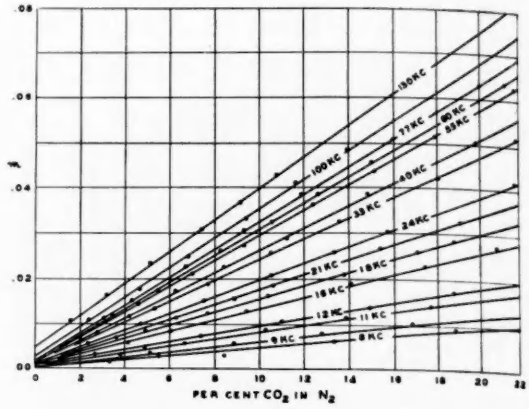

FIG. 5. Graphs showing how the absorption coefficient m depends upon the concentration of CO_2 in N_2.

potassium thiocyanate in a flask cooled to 20°C by means of a water bath. The impurities generated along with the COS were chiefly CS_2 and CO_2. Since the amount of CS_2 present was about 0.01 percent, it was not removed for it was found that this amount would shift the absorption peak of COS only about 6 kc. The CO_2 was removed by slowly passing the COS through KOH solution well filled with glass wool. Traces of SO_2 did not seem to affect the position peak of COS. This was also found true for the effect of SO_2 in CO_2. The COS was dried by passing it through concentrated H_2SO_4 and then over $CaCl_2$ and P_2O_5. Also it was allowed to stand in the purification chambers for two days, during which time the P_2O_5 was shaken at regular intervals.

The purified COS was then displaced into the sound chamber filled with pure N_2; the pressure ratio curves, m graphs, and absorption coefficients per wave-length were obtained for five frequencies. These are shown in Figs. 7, 8, and 9. Only the portion of the low frequency branch of the bell-shaped curve in Fig. 9 was explored because of unavailable higher frequencies.

In order to find the frequency of maximal absorption and to apply Bourgin's theoretical equation (2) a technique has been devised that serves also for investigating the effects of impurities.[3]

The derivation of the equations used in this technique is as follows: It is well known that certain impurities shift the absorption peak of pure absorptive gases to higher frequencies. In most cases the shift will be a linear function of the amount of impurity added. Suppose that μ_2 is the observed value for the absorption per wave-length for the pure gas at frequency ν_2, and μ_1 the absorption at lower frequency ν_1, for the same gas, and that the addition of h percent of an impurity will shift the absorption peak to some higher frequency ν_m'. The peak will be shifted at a rate r defined by $r=\Delta\nu_m'/\Delta h$. Now let h be the percent of added impurity required to reduce the value of μ_2 at frequency ν_2 to the value of μ_1 for the pure gas at frequency ν_1. It can be seen from Eq. (2) that if ν_2 is chosen so that it is very much smaller than ν_m' the denominator of Eq. (2) reduces to $\nu_m'^2$; where $\nu_m'=\nu_m+rh$ is the position of the absorption peak when a percentage h of some impurity has been added to the pure absorptive gas. If ν_2 is higher than ν_1 the absorption equation becomes nearly equal to

$$\mu_1=2\pi\frac{RC_i}{[(C)(C+R)C_\infty(C_\infty+R)]^{\frac{1}{2}}}\frac{\nu_2}{\nu_m+rh}. \quad (5)$$

If one combines (2) and (5), with the above approximations which have been verified experimentally, one has

$$\mu_1=2\pi\frac{RC_i}{[C(C+R)C_\infty(C_\infty+R)]^{\frac{1}{2}}}\frac{\nu_2}{\nu_m+rh}$$

$$=2\pi\frac{RC_i}{[C(C+R)C_\infty(C_\infty+R)]^{\frac{1}{2}}}\frac{\nu_1}{\nu_m}.$$

That is,

$$\nu_m=\nu_1 rh/(\nu_2-\nu_1). \quad (6)$$

In these equations one must keep in mind that a small impurity, such as 1 percent, in a gas like COS will not produce a noticeable error in C_v or C_∞ if its effect is neglected entirely. In a separate investigation[3] it was found that Eq. (6) predicts the position of the frequency of maximal absorption just as accurately as the extrapolated values based upon a complete solution of Eq. (2). Also, it can be seen that

$$\mu_{\max}=\mu_1 rh/2(\nu_2-\nu_1). \qquad (7)$$

As an example, it is found that at 55 kc the addition of water vapor to COS shifts the absorption peak at the rate of 4200 kc for each percent added. Also, at 55 kc (which is ν_2) the value of μ_2 for the pure gas is found to be 0.130 while μ_1 for the same pure gas at 33 kc (which is ν_1) has the value of 0.0825. It is found[3] that 0.0455 percent water vapor added to COS at the frequency of 55 kc reduces the value of μ_2 from 0.130 to the value of 0.0825. From Eq. (6) one has

$$\nu_m=\frac{33\times 4200\times 0.0455}{(55-33)}=287 \text{ kc}.$$

The spectroscopic data given by Bailey[5] were used to calculate the specific heat at constant volume, by using the usual Planck-Einstein equation. The constants at 23°C are: $C_v=7.913$ cal./mole; $C_\infty=5.001$ cal./mole; the sonically activated vibrational heat capacity is calculated to be 2.912 cal./mole, assuming that it includes the first and second levels of the deformation

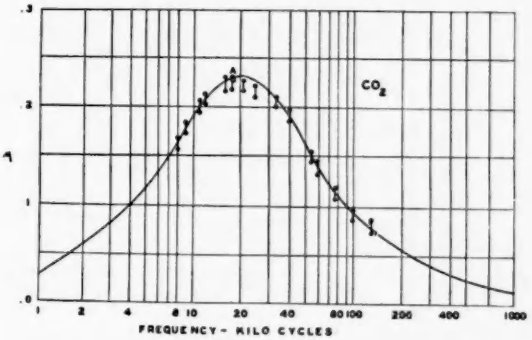

FIG. 6. Curve showing the absorption per wave-length for CO_2. The maximal absorption occurs at 20 kc and μ_{max} has the value of 0.230.

[5] C. R. Bailey and A. B. D. Cassie, Proc. Roy. Soc. London, **A135** (1932).

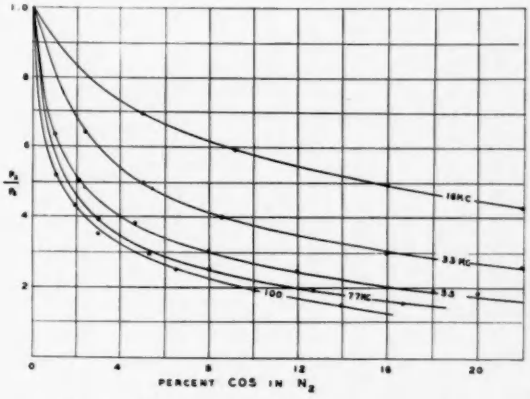

FIG. 7. Curves showing the relative pressures for COS in N_2 for frequencies from 16 to 100 kc.

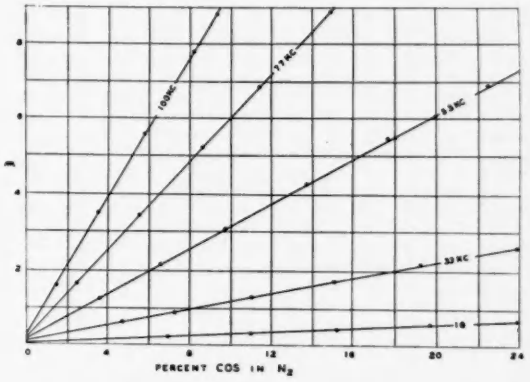

FIG. 8. Graphs showing the absorption coefficient m for COS in N_2.

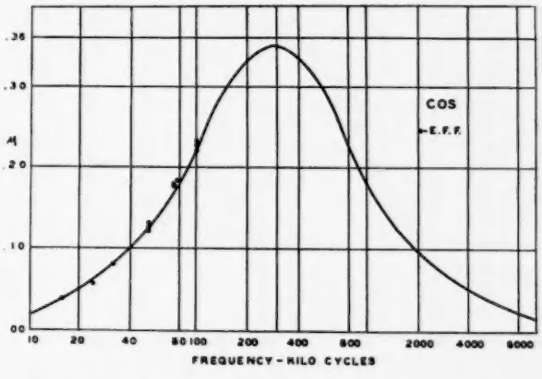

FIG. 9. Curve showing the absorption per wave-length for COS assuming that the maximal absorption occurs at 287 kc.

vibration and the first level of the symmetrical valence vibration. By using Eq. (3) the value of u_{max} is 0.350. The solid curve in Fig. 9 is the theoretical one based on Eq. (2). The average lifetime of the energy quanta is 8.35×10^{-7} sec.

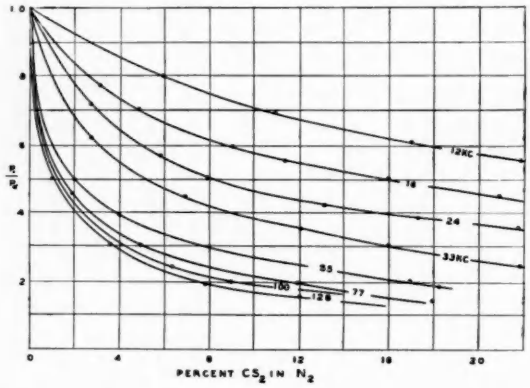

FIG. 10. Curves showing the relative sound pressures for CS_2 in N_2.

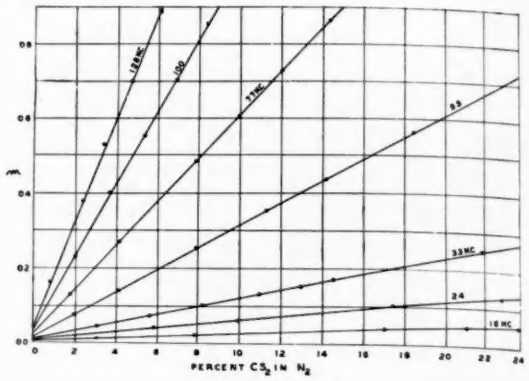

FIG. 11. Graphs showing the absorption coefficient m for CS_2 in N_2.

MEASUREMENTS ON THE ABSORPTION OF SOUND IN CS_2

The liquid carbon disulphide was the purest obtainable from the Braun Corporation, Los Angeles. It had been distilled twice and then dried over P_2O_5.

Known amounts of the liquid were vaporized in the nitrogen in the sound chamber. The ratio curves, m graphs, and resulting values of μ are shown in Figs. 10, 11, and 12.

By using Eq. (6) and different impurities in CS_2 the frequency of maximal absorption is calculated to be 379 kc.[3]

The spectroscopic data given by Dennison,[6] Mecke[7] and Bahgavantam[8] were used to calculate the specific heats: $C_v=9.115$ cal./mole at 23°C; $C_\infty=5.141$ cal./mole; the sonically activated vibrational specific heat is 3.974 cal./

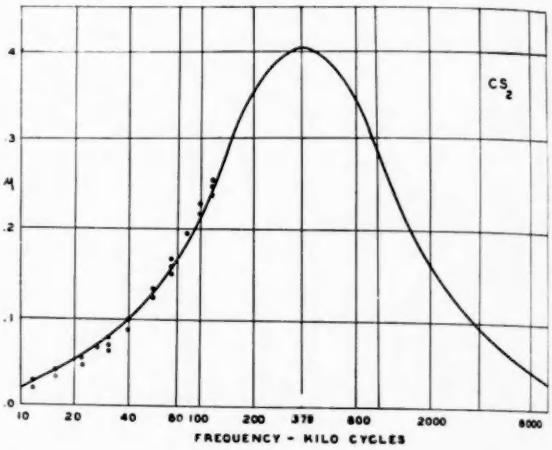

FIG. 12. Curve showing the absorption per wave-length for CS_2 assuming that the maximal absorption occurs at 379 kc.

[6] D. Dennison, Phys. Rev. **38** (1931).
[7] R. Mecke, Zeits. f. phyik. Chemie **409B**, 15–16 (1931).
[8] S. Bahgavantam, Indian J. **7** (1932).

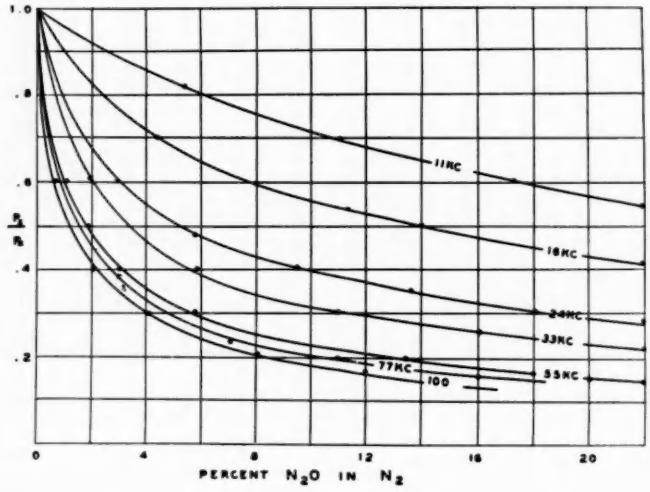

FIG. 13. Curves showing the relative sound pressures for N_2O in N_2.

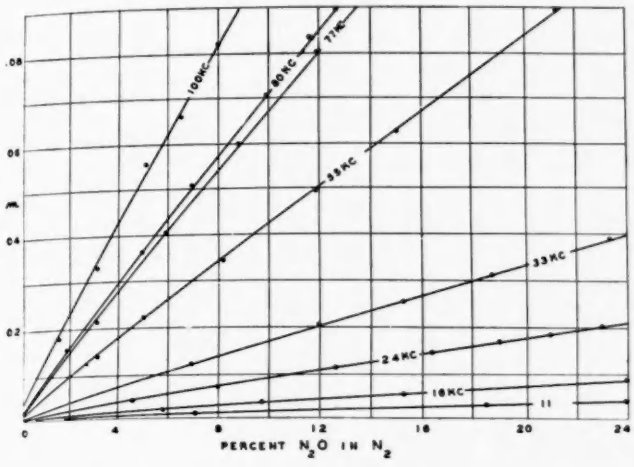

FIG. 14. Graphs showing the absorption coefficient m for N_2O in N_2.

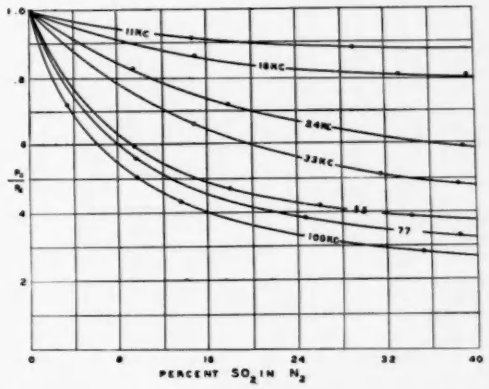

FIG. 16. Curves showing the relative sound pressures for SO_2 in N_2.

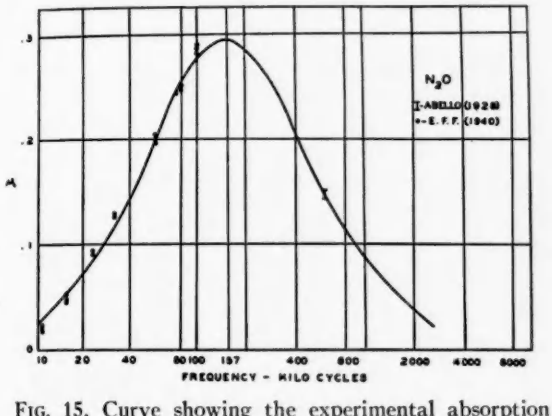

FIG. 15. Curve showing the experimental absorption coefficients μ and the theoretical absorption curve assuming that the maximal absorption occurs at 157 kc.

mole, assuming that the symmetrical valence vibration and the first, second and third levels of the deformation vibration participate. From Eq. (3) the value of μ_{max} is 0.406. The solid curve in Fig. 12 is the theoretical one based on Eq. (2). The lifetime of the energy quanta is 7×10^{-7} second.

MEASUREMENTS ON THE ABSORPTION OF SOUND IN N_2O

The nitrous oxide, in the commercial form, was slowly bubbled through a KOH solution containing glass wool; it was then passed through a $FeSO_4$ solution. These two solutions were designed to remove possible impurities of CO_2 and Cl_2, respectively. The N_2O was then passed over $CaCl_2$ and into the P_2O_5 of the purification chambers that have been described. After remaining in the drying chambers for two days, specimens of the purified N_2O were displaced into the sound chamber containing pure N_2.

The ratio curves for the different percentages of N_2O added to N_2 at seven different frequencies are shown in Fig. 13, and the corresponding m graphs in Fig. 14. The experimental values of μ are shown in Fig. 15.

It was necessary to use the "impurity method" to calculate the frequency of maximal absorption. From an extended investigation,[3] using different impurities, it was found that the frequency of maximal absorption is at 153 kc. This position is considerably lower than that given by Kneser,[9] which was based upon a single point obtained by Abello.[10] However, Kneser used only the deformation vibration in calculating the value of the sonically activated vibrational heat capacity, for use in the theoretical Eq. (2). The experimental point obtained by Abello is shown in Fig. 15.

The spectroscopic data given by Mecke[7] were used to calculate the specific heats: $C_v=7.2427$ cal./mole at 23°C; $C_\infty=4.965$ cal./mole; the sonically activated vibrational specific heat is 2.277 cal./mole, assuming that the first and second levels of the deformation vibration and the symmetrical valence vibration are excited.

The application of these data in Eqs. (3) and (4) shows that $\mu_{max}=0.296$ and the average lifetime of the energy quanta is 14.4×10^{-7} second.

[9] H. O. Kneser, Ann. d. Physik **11**, 761 (1931).
[10] T. P. Abello, Phys. Rev. **31**, 1089 (1928).

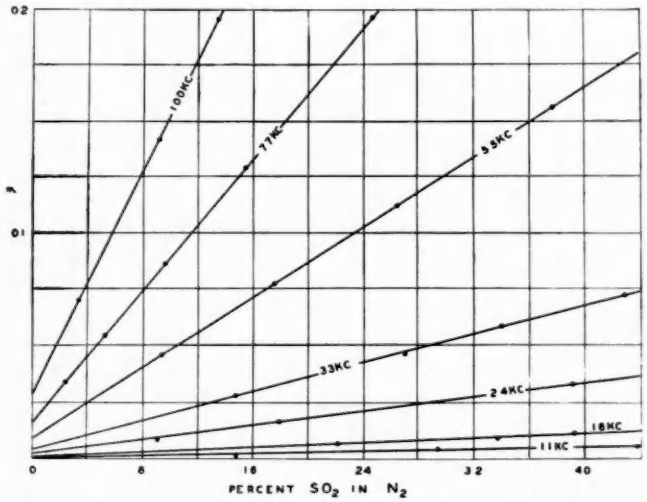

FIG. 17. Graphs showing the absorption coefficient m for SO_2 in N_2.

MEASUREMENTS ON THE ABSORPTION OF SOUND IN SO_2

The sulphur dioxide was commercial gas obtained from the Ohio Chemical Company. The gas was slowly passed through concentrated sulphuric acid and then stored over $CaCl_2$ and P_2O_5. The gas thus prepared was added to pure N_2 in the sound chamber. The values of the ratio curves are shown in Fig. 16, the m graphs in Fig. 17, and the usual μ curve in Fig. 18.

The frequency of maximal absorption was determined by the method of impurities, adding small amounts of methyl alcohol as a catalyst. The position of the absorption peak was found to be at 1040 kc.

The spectroscopic data were obtained from Badger and Bonner.[11] The calculated values of the specific heats at 23°C are: $C_v = 7.5244$ cal./mole; $C_\infty = 6.1084$ cal./mole; the sonically activated vibrational specific heat is 1.416 cal./mole, assuming that only the two lowest fundamental vibrations are active.

CONCLUSIONS

In this study of the absorption of sound in CO_2 it is seen that the absorption maximum is at 20 kc. It appears from the value of μ_{max} that the symmetrical valence vibration is not active at low frequencies of sound, although it may become active at higher frequencies.

[11] R. M. Badger and L. G. Bonner, Phys. Rev. **43**, 305 (1940).

The intensity method for absorption measurements has two inherent sources of error: the one from the uncertainty in determining the absorption of the walls of the sound chamber; the other from the measurement of the amount of added gas. It is probable that the first source introduces an error in m of approximately two percent, but this source of error requires further investigation. However, it would require a twelve percent error in the determination of $\alpha S/4V$ (as given in Eq. (1)) to make the experimental values of m, and thus μ, so indefinite that the symmetrical valence vibration of CO_2 would escape detection if it were excited by low frequency sound waves. The second source of error above was made negligible through the procedure of using different amounts of CO_2 in N_2 ranging from O to 100 percent CO_2.

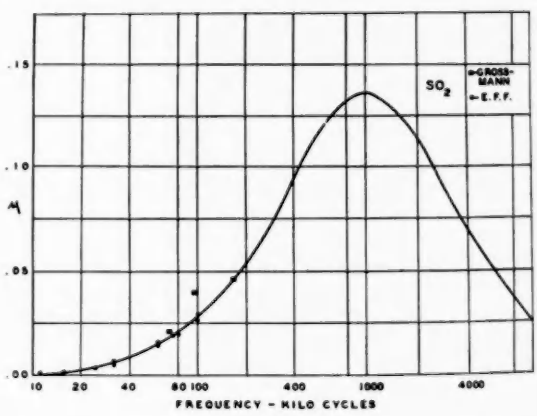

FIG. 18. Curve showing the absorption per wave-length. The solid curve is the theoretical one based upon the assumption that the maximal absorption occurs at 1040 kc.

The procedure also eliminated the possibility that N_2 obeys an absorption law other than the classical one. Further, the region of maximum absorption of CO_2 occurs at a frequency so low that 100 percent of CO_2 could be used for the determination of the absorption maximum. The discrepancy between the absorption measurements on CO_2 by the intensity method, here used, and by the direct method, used by Leonard, has not yet been explained, but is the object of further investigation in this laboratory.

From a comparison of the four linear molecules CO_2, N_2O, COS, and CS_2, it may be inferred that the absorption maximum of CO_2 occurs at a relatively low sound frequency because it is a characteristic of the deformation vibration to have energy quanta with a long lifetime. For gases with lower fundamental vibrational modes than CO_2 it is seen that the absorption maxima occur at higher frequencies of sound, as is the case of N_2O, COS, and CS_2, in the order named. However, it has not yet been possible to explain why a straight line results when the values of μ_{max} are plotted against the corresponding frequencies of maximal absorption for these linear molecules. The linear relation is shown in Fig. 19. (In Fig. 20 are shown the corresponding values of m plotted against the frequencies of maximal absorption.) From Fig. 19 it may be inferred that the frequencies of the absorption maxima of the four other known triatomic linear molecules are: HCN at 18 kc, ClCN at 380 kc, BrCN at 435 kc, and ICN at 460 kc.

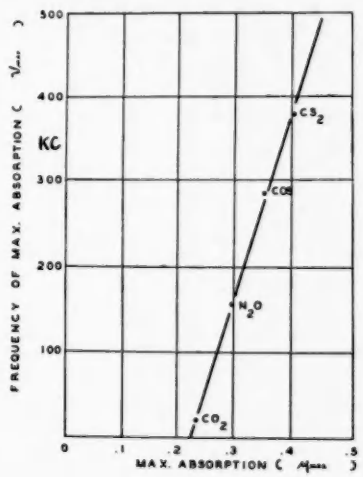

FIG. 19. Graph showing the relationship between the frequency of maximal absorption and the maximal absorption coefficient μ_{max} for four triatomic linear molecules.

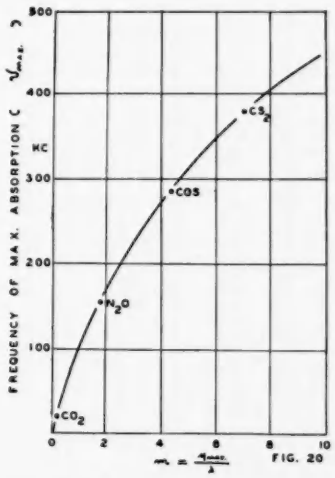

FIG. 20. Curve showing the relationship between the frequency of maximal absorption and the absorption coefficient m. The magnitude of m is the value m would have for the particular gas at the frequency of maximal absorption.

The occurrence of the absorption peak of the nonlinear SO_2 molecule at 1040 kc is possibly attributable to its low fundamental frequencies of vibration. This suggests that there are other triatomic nonlinear molecules, such as H_2S and H_2O, that have frequencies of maximal absorption lower than that of SO_2. It is also apparent that ClO_2 should have a peak occurring at a frequency much higher than that of SO_2 because of its lower fundamental frequencies.

From the theories of collision probabilities and quantum transitions[12] it will be recalled that molecules which have their maximal absorption at low sound frequencies require many more collisions to deactivate them than do those which occur at higher frequencies. This can be seen in Table I. In Table I the symbols have the following meanings: d_a is the molecular diameter; S is Sutherland's constant; ν_m is the frequency of maximal absorption; β is the lifetime of the energy quanta; k_{10} is the reaction rate from the excited to the normal state; P_{1_0} is the probability of the removal of one quantum of vibration from an excited molecule; B_{10} is the number of collisions required to remove a quantum of

[12] A. Eucken, Zeits. f. physik. Chemie **B27**, 235 (1934); **B41**, 199 (1938).

TABLE I. *Summary of collision phenomena.*

GAS	CO_2	N_2O	COS	CS_2	SO_2
$d_a \times 10^8$	3.2	3.2	3.73	3.9	3.3 cm
S	274	274	330	401	416
ν_m kc	20	157	287	379	1040
$\beta \times 10^7$	102.6	14.4	8.6	7.0	1.81
$k_{10} \times 10^{-5}$	0.935	6.75	10.6	12.6	52.4
$P_{10} \times 10^5$	1.16	8.40	10.4	11.5	52.8
B_{10}	86,000	11,800	9,600	8,700	1,900
$W \times 10^{-8}$	0.52	3.9	3.92	3.91	20

energy; W is the number of quantum transitions per second.

There have been many investigations and discussions on the absorption of sound in some of these gases. Most of the findings show that the symmetrical valence vibrations are excited by sound waves. They also show that the symmetrical valence vibration has a shorter lifetime than the deformation vibration. These findings are discussed by Richards and Reid,[13] Wallmann,[14] Eucken,[12] Itterbeek,[15] and Pielemeier.[16] Richards' value of μ_{max} for CO_2 agrees more closely with that obtained in this investigation than do those obtained by others. The position of the absorption peak found by Itterbeek is nearly identical with that found in this investigation.

The author wishes to thank Dr. V. O. Knudsen for his interest, guidance, and valuable suggestions over the period devoted to the completion of this dissertation. Acknowledgment also is made to the University of California at Los Angeles for valuable fellowships.

[13] W. T. Richards and J. A. Reid, J. Chem. Phys. **1**, 863 (1933); **1**, 737, 144 (1933); **2**, 193, 206 (1934).
[14] Marie-Helene Wallmann, Ann. d. Physik **21**, 671 (1934).
[15] A. Van Itterbeek, Physica **7**, 128 (1940).
[16] W. H. Pielemeier, J. Chem. Phys. **8**, 106 (1940).

The Absorption of Sound in CO_2, N_2O, COS, and in CS_2, Containing Added Impurities

V. O. KNUDSEN AND E. FRICKE
University of California at Los Angeles, Los Angeles, California

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We have investigated the absorptive characteristics of CO_2, N_2O, COS, and CS_2 as influenced by the addition of certain gases, such as H_2, H_2O, H_2S, CH_3OH, etc., acting as "catalysts." These catalysts shift the absorption bands to higher frequencies; the magnitudes of these shifts yield information respecting (1) the frequency at which each pure gas has its maximal absorption, and (2) the nature of the molecular collisions involved, and especially the effectiveness of these collisions in disturbing the vibrational states of the absorptive molecules. The results indicate that at atmospheric pressure and 23°C the absorption maxima for pure CO_2, N_2O, COS, and CS_2 are shifted in each case by amounts proportional to the concentration of the added catalyst. The addition of one percent of H_2O to CO_2 shifts the absorption band for CO_2 2250 kc; similarly, one percent of H_2O shifts the CS_2 band 2460 kc, the COS band 4200 kc, and the N_2O band 427 kc. One percent of the other impurities produced shifts which varied from 20 to 1830 kc. Transition probabilities for the above gases have been calculated. These probabilities are characteristic of the colliding pair of molecules.

THE writers have determined the absorptive characteristics of CO_2, N_2O, COS, and CS_2—all linear molecules—as influenced by the addition of small amounts of certain other gases, as impurities. The impurities selected for this study were H_2, H_2O, H_2S, CH_3OH, C_3H_7OH, and $C_6H_5CH_3$. The four primary gases, CO_2, N_2O, COS, and CS_2, which will be designated as A molecules, are highly absorptive for sound; each has a characteristic absorption band. The frequencies of these bands or peaks are peculiarly dependent upon the kinds and amounts of added impurities. These impurities will be designated as B molecules. All the above-named B molecules shift the absorption bands of the A molecules to higher frequencies. This is interpreted, according to well-established absorption theory, as a decrease in the time of survival of activated (usually vibrating) A molecules; i.e., collisions between A and B molecules (AB collisions) are more effective in dissipating the activated A molecules than are AA collisions. Since the above named impurities increase the reaction rates of transitions between the vibrating and nonvibrating A molecules, these impurities are sometimes called "catalysts." It is well known that H_2O, for example, has a very large "catalytic" effect in speeding up the transition reaction rate for CO_2.

The apparatus and technique of measurement used in the present investigation are based upon the intensity method of sound absorption,[1] but both apparatus and technique have been improved.[2] The absorption chamber, a brass-walled cylinder 21 cm in diameter and 30 cm long, is described in the previous paper.[2] The method of measuring the sound pressure in the chamber and calculating the absorption coefficient is essentially the same as the one described earlier.[1,2]

The method of determining the rate of shift of the absorption peak of a primary type A gas, as increasing amounts of the catalytic type B gas are added, and also the frequency at which the pure type A gas will have its maximal absorption, is based on measurements of the absorption coefficients μ_1 and μ_2 of the pure A gas at two different frequencies ν_1 and ν_2, and on the amount of impurity h which, at frequency ν_2, must be added to gas A in order to reduce μ_2 to the value of μ_1. The equations are presented in the preceding paper.[2] The equation for calculating the frequency of the maximal absorption for the pure gas A is

$$\nu_m = \nu_1 r h/(\nu_2 - \nu_1), \qquad (1)$$

where r is the shift of the absorption peak, in kilocycles per percent of added impurity.

It will be recalled from the general Bourgin

[1] V. O. Knudsen and E. Fricke, J. Acous. Soc. Am. **10**, 89 (1938).
[2] E. Fricke, this journal, preceding paper, p. 245.

equation

$$\mu = 2\pi \frac{RC_i}{[C(C+R)C_\infty(C_\infty+R)]^{\frac{1}{2}}} \frac{\nu_m \nu}{\nu_m^2+\nu^2} \qquad (2)$$

that if, at a constant frequency ν, h percent of an impurity be added to gas A the frequency of the maximal absorption will be shifted to some higher frequency ν_m', given by

$$\nu_m' = \nu_m + rh. \qquad (3)$$

When working on the low frequency branch of the absorption curve at a certain frequency ν, the value of μ will continue to decrease as an impurity B is added to the gas A. It is then necessary only to measure the corresponding values of μ which result when different amounts of the impurity B has been added to the gas A. A complete solution of Eq. (2) for the unknown frequency of maximum absorption ν_m', since all the other terms may be calculated from band spectra data and sound measurements, will yield data which obey the linear relation between ν_m' and h. For the pure gas, the line represented by Eq. (3) intersects the frequency axis at ν_m. (See, for example, Figs. 3 to 6.) Equation (1) yields the value of ν_m directly. However, it is limited to gas mixtures of the type under investigation. For example, this equation cannot be employed to calculate the frequency of maximum absorption of a gas such as CO_2 by adding H_2 as an impurity. For this mixture one must make a complete solution of Eq. (2). However, experimental evidence shows that Eq. (1) holds for a gas mixture such as CO_2 and water vapor, since so little water vapor as an impurity is required to produce significant shifts of the absorption band.

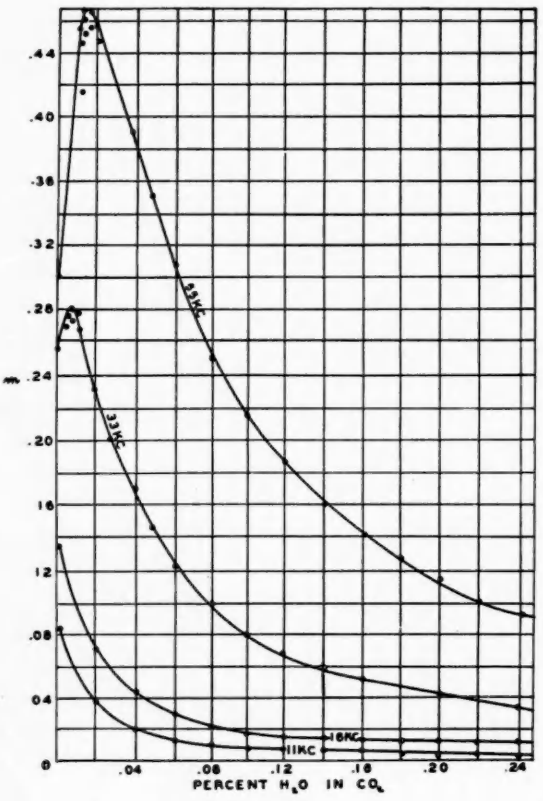


FIG. 1. The absorption coefficient m, per cm, of CO_2 containing small amounts of water vapor, for frequencies of 11, 16, 33, and 55 kc.

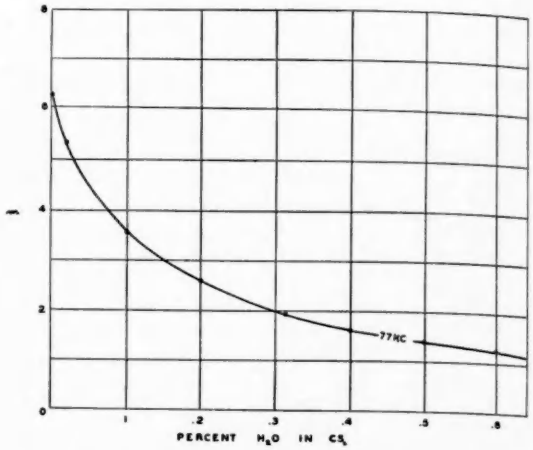


FIG. 2. The absorption coefficient m, per cm, of CS_2 containing small amounts of water vapor, for a frequency of 77 kc.

From the experiments, based on Eqs. (1) and (2), and using several impurities and frequencies, the values of ν_m for the four primary gases are as follows:

CO_2	N_2O	COS	CS_2
20 kc	153 kc	287 kc	379 kc.

From these values of ν_m the characteristic lifetimes B_{AA} may be calculated.

In Figs. 1 to 6 are shown the experimental results of this study on the effects of certain impurities on the absorption of sound in CO_2, N_2O, COS, and CS_2. Table I gives a typical set of data for CO_2 containing H_2O as the impurity for frequencies of 11, 16, 33, and 55 kc. Figure 1 is a graphical record of the data in Table I but includes additional data for 33 and 55 kc in the

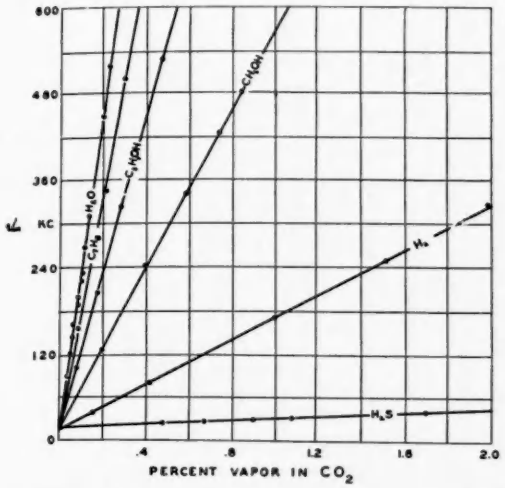


FIG. 3. The frequency of maximal absorption of CO_2 containing small amounts of the indicated impurities.

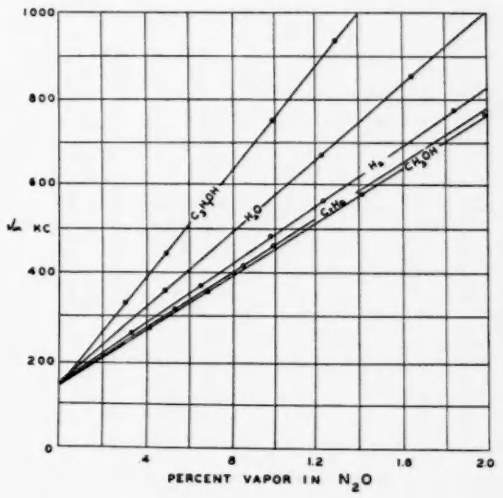


FIG. 4. The frequency of maximal absorption of N_2O containing small amounts of the indicated impurities.

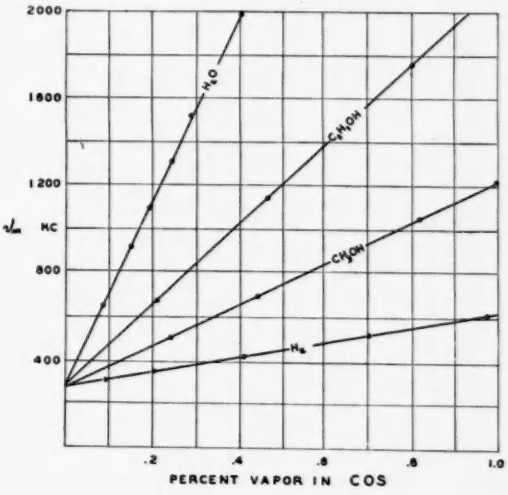


FIG. 5. The frequency of maximal absorption of COS containing small amounts of the indicated impurities.

regions of greatest absorption. Figure 2 is a similar curve for H_2O in CS_2 for a frequency of 77 kc.

The values of r, the shift of the absorption peak in kc for each percent of added impurity, for all gases investigated in this series, are summarized in Table II. These values of r are obtained from the slopes of the straight lines of Figs. 3 to 6, which show the dependence of the ν_m' on the impurities present in each of the four primary gases. The straight lines of Figs. 3 to 6 indicate that r is a constant, independent of the sound frequency and characteristic of the pair of colliding molecules, as CO_2/H_2, CO_2/H_2O, etc. Note, for example, the striking contrast in the effects of H_2O and H_2S on the absorption of sound in CO_2.

In order to calculate the number of collisions of the AB type (that is, collisions between the primary molecule A and the "catalyst" molecule B) per unit time, it is necessary to know, among other things, the diameters of the colliding molecules. Data relating to the diameters of the molecules here considered are summarized in Table III.

Finally, in Table IV are given certain significant molecular data which have been calculated from the experimental results of the present investigation. The first column gives the pair of molecules concerned, that is, CO_2/H_2 means that the primary gas is CO_2, and the impurity is H_2.

β^{AB} is the characteristic lifetime of the molecule A provided all its collisions are with B molecules, and is defined by

$$\beta^{AB}=\frac{\beta^{AA}\beta(f)}{\beta^{AA}-\beta(1-f)},$$

where f is the fractional amount of the impurity B in the gas mixture, β is the characteristic lifetime for the energy quanta of the absorptive gas in the gas mixture, defined by

$$\beta=\frac{1}{2\pi\nu_m'}\left(\frac{C(C+R)}{C_\infty(C_\infty+R)}\right)^{\frac{1}{2}},$$

where C is the specific heat at constant volume (cal./mole) for the gas mixture, and C_∞ is the specific heat at constant volume (cal./mole) for that portion of the gas mixture which is independent of the sound cycle.

$k_{10}{}^{AB}$ is the specific reaction rate from the activated to the normal state of molecule A resulting from collisions with molecule B, and is approximately defined by

$$k_{10}{}^{AB}=(1-e^{-\theta/T})^{-1}/\beta^{AB}$$

where θ is the characteristic temperature of the deformation vibration of molecule A, and T is the absolute temperature (296°K in these experiments).

$P_{10}{}^{AB}$ is the probability that a vibrating molecule A colliding with a molecule B will lose its vibrational energy, and is defined by

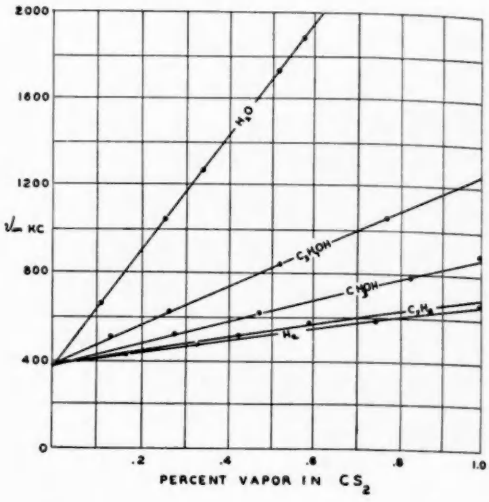


FIG. 6. The frequency of maximal absorption of CS_2 containing small amounts of the indicated impurities.

$$P_{10}{}^{AB}=\frac{k_{10}{}^{AB}}{2\pi RT\left(\frac{M_A+M_B}{M_AM_B}\right)^{\frac{1}{2}}\left(1+\frac{S_{AB}}{T}\right)\left(\frac{d_A+d_B}{2}\right)^2 A},$$

where A is the number of molecules per cubic centimeter at temperature T (296°K); d_A and d_B the diameters of the A and B molecules; M_A and M_B the molecular weights of the A and B molecules; R the gas constant; and $S_{AB}=(S_AS_B)^{\frac{1}{2}}$ is Sutherland's constant for the gas mixture, where S_A and S_B are Sutherland's constants for gases A and B, respectively.

$B_{10}{}^{AB}$ is the number of collisions the activated A molecule must make, on the average, with B molecules in order to become deactivated, and is simply the reciprocal of $P_{10}{}^{AB}$. It will be seen

TABLE I. CO_2/H_2O.

h (%)	ν_m' (KC)	m (11 KC)	m (16 KC)	m (33 KC)	m (55 KC)
0	20	0.082	0.135	0.255	0.300
0.008	28			.282	
.02	65	.038	.070	.232	.465
.04	122	.0208	.0424	.170	.390
.06	160	.0142	.0313	.123	.308
.10	248	.0092	.0196	.082	.216
.12	296	.0076	.0162	.068	.185
.14	338	.0066	.014	.059	.160
.16	386	.0057	.0121	.051	.140
.18	.430	.0052	.0116		.128
.20	470	.0047	.010	.042	.115
.22	518	.0042	.009	.039	.103
.24	560	.0039	.0083	.035	.0963
.26	602				.0911
.28	640			.030	.0840
.30	692			.028	.07804

TABLE II. *Shifts of absorption peak (kc/percent) for impurities.*

PRIMARY GAS	H_2	H_2O	H_2S	CH_3OH	C_3H_7OH	$C_6H_5CH_3$
CO_2	155	2250	20.5	550	1100	1780
N_2O	342	427	—	304	575	300
CS_2	296	2460	—	496	871	316
COS	336	4200	—	930	1830	—

TABLE III. *Data for calculating molecular diameters:* $d=4.56\times10^{-20}(\sqrt{M/\eta})(1/(1+S/273))$, *$d$ = molecular diameter, η = viscosity coefficient, M = molecular weight, S = Sutherland's constant.*

MOLECULE	M	η	S	d
CO_2	44	145×10^{-6}	274	3.2×10^{-8} cm
N_2O	44	144	274	3.2
CS_2	76	96	401	3.9
COS	60	110	330	3.73
SO_2	64	228	416	3.38
H_2	2	85	80	2.47
H_2O	18	214	650	2.72
H_2S	34	147	331	3.12
CH_3OH	32	135	475	3.7
C_3H_7OH	60	162	600	4.3
$C_6H_5CH_3$	92	81	735	5.3

TABLE IV.

GAS	β^{AB} (SEC.)	$k_{10}{}^{AB}$	$P_{10}{}^{AB}$	$B_{10}{}^{AB}$
CO_2/H_2	12.65×10^{-9}	7.6×10^{7}	4.65×10^{-3}	215
N_2O/H_2	6.49×10^{-9}	1.46×10^{8}	9.1×10^{-3}	110
CS_2/H_2	9.02×10^{-9}	9.7×10^{7}	4.5×10^{-3}	220
COS/H_2	6.06×10^{-9}	1.65×10^{8}	7.7×10^{-3}	130
CO_2/H_2O	9.8×10^{-10}	9.8×10^{8}	6×10^{-2}	17
N_2O/H_2O	4.78×10^{-9}	1.98×10^{8}	1.76×10^{-2}	57
CS_2/H_2O	10×10^{-10}	8.7×10^{8}	5.85×10^{-2}	18
COS/H_2O	5.4×10^{-10}	1.72×10^{9}	1.28×10^{-1}	8
CO_2/H_2S	8.7×10^{-8}	1.1×10^{7}	8.3×10^{-4}	1200
CO_2/CH_3OH	3.2×10^{-9}	3×10^{8}	2.8×10^{-2}	36
N_2O/CH_3OH	7.74×10^{-9}	1.23×10^{8}	1.1×10^{-2}	95
CS_2/CH_3OH	5.15×10^{-9}	1.7×10^{8}	1.2×10^{-2}	83
COS/CH_3OH	2.54×10^{-9}	3.64×10^{8}	2.7×10^{-2}	37
CO_2/C_3H_7OH	1.94×10^{-9}	4.95×10^{8}	3.94×10^{-2}	25
N_2O/C_3H_7OH	3.74×10^{-9}	2.54×10^{8}	$2.\times10^{-2}$	50
CS_2/C_3H_7OH	2.79×10^{-9}	3.1×10^{8}	2.1×10^{-2}	48
COS/C_3H_7OH	1.57×10^{-9}	5.9×10^{8}	4.16×10^{-2}	24
$CO_2/C_6H_5CH_3$	1.2×10^{-9}	8×10^{8}	4.1×10^{-2}	25
$N_2O/C_6H_5CH_3$	6.8×10^{-9}	1.4×10^{8}	8.9×10^{-3}	113
$CS_2/C_6H_5CH_3$	9.9×10^{-9}	8.8×10^{7}	5×10^{-3}	200

from the last column of Table IV that the effectiveness of collisions to disturb the vibration of a molecule depends significantly upon the *pair* of colliding molecules. Thus, a vibrating COS molecule will survive, on the average, only 8 collisions with H_2O molecules, whereas a vibrating CO_2 molecule survives 1200 collisions with H_2S molecules. For comparison, a vibrating CO_2 molecule will survive 86,000 collisions with other CO_2 molecules. Similarly, the corresponding number of collisions $B_{10}{}^{AA}$ is 11,800 for N_2O, 9600 for COS, and 8700 for CS_2.

Data of the type here presented should aid the theoretical physicist and chemist in gaining a better understanding of certain molecular collisions and reactions.

Symmetrical Sand Figures on Circular Plates

R. C. COLWELL, J. K. STEWART AND H. D. ARNETT
Department of Physics, West Virginia University, Morgantown, West Virginia
(Received March 10, 1940)

It is shown that Kirchhoff's solution for a circular plate is a special case which gives circles and diameters only. If this solution is generalized, the very intricate figures actually found upon circular plates may be calculated mathematically. In particular, a few figures found by Chladni are computed from the new equations. The methods of computation from the curves of the Bessel functions are outlined briefly.

RAYLEIGH'S well-known solution for the vibration of a circular plate is

$$w_n = P\cos(n\theta-\alpha) \times\{J_n(kr)+\lambda J_n i(kr)\}\cos(pt-\epsilon). \quad (1)$$

When this is equated to zero, the resulting values give the position of the nodal lines, which for this equation are always circles and diameters.

McMahon[1] has shown that the pth root of $J_n(x)=0$ is approximately

$$x=\beta-\frac{m-1}{4\beta}-\frac{4(7m^2-38m+31)}{3(8\beta)^3},$$

where

$$\beta=\pi(2n+4s-1)/4 \quad \text{and} \quad m=4n^2$$

and the sth root of $J_n'(x)=0$ is

$$x=\gamma-\frac{m+3}{8\gamma}-\frac{4(7m^2+82m-9)}{3(8\gamma)^3},$$

where

$$\gamma=\pi(2n+4s+1)/4 \quad \text{and} \quad m=4n^2.$$

These curves intersect at a point halfway between their corresponding roots, providing their succeeding maxima are nearly equal, that is when

$$x=\frac{\pi}{2}(n+2s)-\frac{4(112n^4+88n^2+11)}{3[4\pi(n+2s)]^3}. \quad (2)$$

From Eq. (2), values of ka are calculated for Kirchhoff curves consisting of s circles and n diameters. (See Table I.)

However, in addition to circles and diameters, many other symmetrical figures appear upon the circular plates. Chladni attributed these to irregularities in the plates. Rayleigh suggested that they were caused by the manner in which the plates were supported. Other factors such as the crystalline structure of the rolled metal and the dimensions of the central clamp led investigators to conclude that the discrepancies were due to experimental difficulties and not to the theory. But with more delicate and powerful methods of vibrating the plates, the curves instead of falling into the circular and diametral patterns given by the Kirchhoff solution, tend even more to form complex and intricate (but still symmetrical) designs.

The conclusion of Kirchhoff's theory that two modes cannot exist simultaneously on a vibrating plate, is inherent in his hypothesis and his solution is a particular one based upon that assumption. Rayleigh[2] points out that "it may be that under the action of the bow, two or more normal component vibrations exist."

In this paper, it will be shown how the circles and diameters of an ideal plate are plotted from Eq. (1) with the use of the general equation rather than the approximation which holds only when ka is much greater than n. The nodal

TABLE I. *Values of ka for various nodal systems, n diameters; s circles.*

S	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
0	——	——	2.292	3.497	4.65	5.75
1	3.014	4.530	5.937	7.274	8.55	9.79
2	6.209	7.737	9.16	10.55	11.95	13.23
3	9.370	10.91	12.41	13.86	15.24	16.57
4	12.53	14.08	15.58	17.05	18.45	19.81
5	15.68	17.23	18.73	20.21	21.63	23.01

[1] A. Gray, G. B. Matthews and T. M. MacRobert, *Bessel Functions* (Macmillan Company, 1922).

[2] Lord Rayleigh, *Sound*, Vol. I (Macmillan, second edition, 1894), p. 368.

systems formed by two coexistent modes on the plate will also be investigated and it will be demonstrated that all the figures found by Chladni can be accounted for by two or more coexistent vibrations.

The behavior of the Bessel function of the first kind is well known and its zeros have been tabulated.[1] The function is defined by

$$\left.\begin{aligned} J_n(ikr) &= \sum (-1)^p (ikr)^{n+2p}/2^{n+2p}(p)!(n+p)! \\ &= i^n \sum (kr)^{n+2p}/2^{n+2p}(p)!(n+p)! \\ &= i^n I_n(kr). \end{aligned}\right\} \quad (3)$$

Substitution of this equation in Eq. (1) gives

$$w_n = P\cos(n\theta-\alpha) \times\{J_n(kr)+\lambda i^n I_n(kr)\}\cos(pt-\epsilon). \quad (4)$$

The first and second derivatives of this with respect to r will give

$$\left.\begin{aligned} &\frac{d}{dr}J_n(kr) = kJ_{(n-1)}(kr) - \frac{n}{r}J_n(kr), \\ &\frac{d^2}{dr^2}J_n(kr) = \left[\frac{n^2+n}{r^2}\right]J_n(kr) - \left[\frac{2nk-k}{r}\right]J_{n-1}(kr) + k^2 J_{(n-2)}(kr), \\ &\frac{d}{dr}J_n(ikr) = ikJ_{(n-1)}(ikr) - \frac{n}{r}J_n(ikr), \\ &\frac{d^2}{dr^2}J_n(ikr) = \left(\frac{n^2+n}{r^2}\right)J_n(ikr) - \left[\frac{2nki-ik}{r}\right]J_{n-1}(ikr) - k^2 J_{n-2}(ikr). \end{aligned}\right\} \quad (5)$$

Now the boundary conditions for a circular plate with free edge are known to be

$$\left.\begin{aligned} &\frac{d}{dr}\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right) - n^2\left(\frac{2-\mu}{a^2}\frac{dw}{dr} - \frac{3-\mu}{a^3}w\right) = 0, \\ &\frac{d^2w}{dr^2} + \mu\left(\frac{1}{a}\frac{dw}{dr} + \frac{1}{a^2}\frac{d^2w}{d\theta^2}\right) = 0. \end{aligned}\right\} \quad (6)$$

Equations (5) and (6) together will give

$$-\lambda i^n = \frac{R_1 J_n(ka)+R_2 J_{(n-1)}(ka)+R_3 J_{(n-2)}(ka)}{R_1 I_n(ka)+R_2 I_{(n-1)}(ka)+R_3 I_{(n-2)}(ka)}, \quad (7)$$

where

$$\left.\begin{aligned} R_1 &= (1-\mu)(n^2+n), \\ R_2 &= ka(1-2n+\mu), \\ R_3 &= k^2a^2. \end{aligned}\right\} \quad (8)$$

TABLE II. *Values of* λ *for certain nodal systems,* n *= diameters;* s *= circles.*

S	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
0	—	—	−0.257	+0.0766i	+0.0439	−0.0223i
1	−0.0808	+0.0190i	+0.00668	−0.00187i	−0.000364	+0.000199i
2	+0.0041	−0.0007i	−0.00018	+0.000057i		
3	−0.00013					

With the values of (ka) from Table I and Eq. (7), it is possible to evaluate λ for the various nodal systems. These values are shown in Table II.

From Table II it appears that as ka becomes larger, the values of λ become vanishingly small. The nodal circles are then given by $J_n(kr)=0$ as in the case of membranes. As ka becomes larger, the frequency increases so that for high frequencies the nodal systems may be calculated with λ equal to zero.[3] For small values of ka, the behavior of the function $I_n(kr)$ must be considered. This is best done by plotting the curve $y=I_n(x)$ for various values of n, which is shown in Fig. 1. To find the curves represented by Eq. (1) for one circle and diameter, it is permissible to write

$$\cos\theta[J_1(kr)+\lambda i I_1(kr)]=0, \quad (9)$$

where

$$k=4.53/a, \quad \lambda=0.019i$$

and a is the radius of the plate. The factor $\cos\theta$ accounts for one diameter, while the second

[3] R. C. Colwell and H. C. Hardy, Phil. Mag. **24**, 1041–1055 (1937).

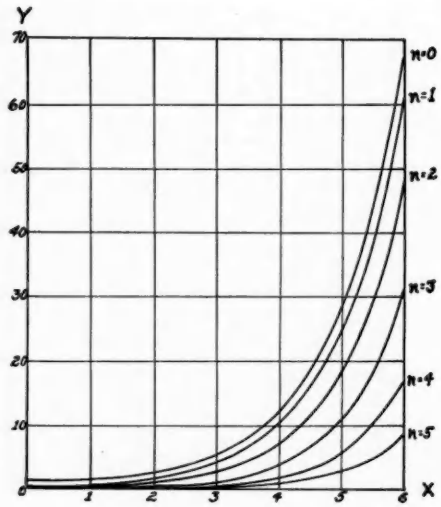


FIG. 1. Values of the function $y=I_n(x)$.

factor is plotted by addition of coordinates as shown in Fig. 2. The curve b represents the graph of $y=J_1(kr)+\lambda iI_1(kr)$ from which roots of r/a may be read by intersections with the horizontal axis. Figure 3 is plotted from these values.

When two modes of vibration are coexistent upon the same plate, the equation of the nodal lines takes the form

$$w=A[\cos n(\theta-\alpha_1)\{J_n(kr)+\lambda J_n(ikr)\}] +B[\cos m(\theta-\alpha_2)\{J_m(kr)+\lambda J_m(ikr)\}] \quad (10)$$

in which the ratio of A to B varies from negative infinity to positive infinity, according as the first or second mode is predominant.[4]

For example, consider a plate which is set in motion in such a way that the resulting nodal system is intermediate between the Kirchhoff solution which has two circles and two diameters

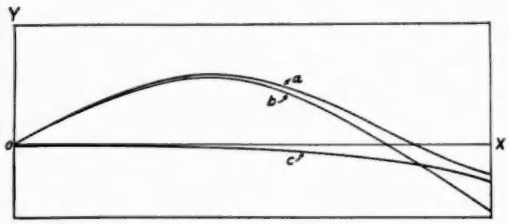


FIG. 2. Curve a represents $J_1(x)$; curve b represents $J_1(x)+\lambda I_1(x)$; curve c is $I_1(x)$.

with the equation

$$\left[J_2\left(\frac{9.16r}{a}\right)+0.00018I_2\left(\frac{9.16r}{a}\right)\right]\cos 2\theta=0 \quad (11)$$

and that consisting of three circles and one diameter with the equation

$$\left[J_1\left(\frac{10.91r}{a}\right)\right]\cos\theta=0. \quad (12)$$

The value of λ is neglected in Eq. (12).

The addition of Eq. (11) and Eq. (12) gives the form of Eq. (10) so that

$$J_2=kJ_1 \quad (13)$$

in which $J_n=J_n(kr)+\lambda J_n(ikr)$ and $k=-B\cos\theta/A\cos 2\theta$. For any ratio $-B/A$, the values of k may be determined for all the values of θ from 0° to 360°, or from 0° until the values start to repeat. Now the curves $y=J_1$ and $y=J_2$ are plotted for each value of k. These curves intersect in points whose abscissas are the values of r, which satisfy Eq. (13). Figure 4 shows these curves for $B=A$, $\theta=0°$, $k=1$. From the graph, there are two points $\theta=0°$, $r=0.441a$, $r=0.688a$.

Many investigators have found intricate patterns on circular plates for which no mathe-

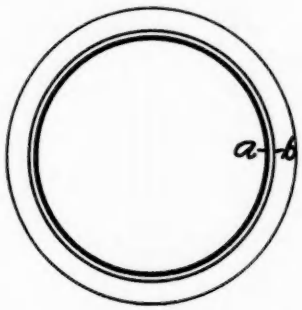


FIG. 3. The light curve b represents $J_1(x)$. The heavy curve a shows the correction due to λ.

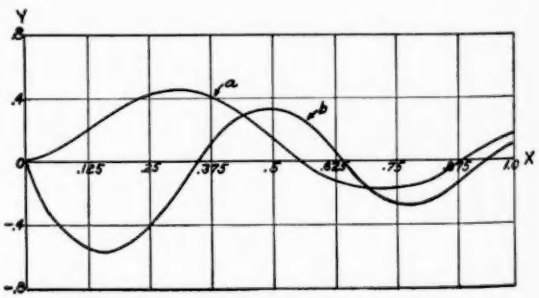


FIG. 4.

[4] J. K. Stewart and R. C. Colwell, J. Acous. Soc. Am. **11**, 147–151 (1939).

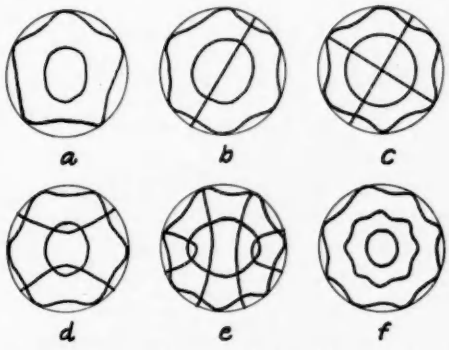


FIG. 5. Line drawings from Chladni's figures. The irregularities are due to non-uniformities in the plates.

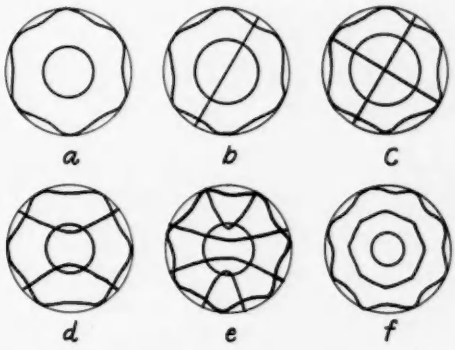


FIG. 6. The mathematical figures corresponding to Chladni's experimental ones.

matical equations have been worked out. Several typical formulas for these will now be given. In Chladni's *Akoustik*, the following plates are found by experiment (Fig. 5). They are taken from an article by Waller.[5]

A pattern similar to Fig. 5a is obtained by combining two circles and no diameters with no circles and six diameters (Fig. 6a). The exact pattern would be obtained by using five diameters instead of six. The equation is

$$8J_0\left(\frac{6.209r}{a}\right)+5J_6\left(\frac{6.80r}{a}\right)\cos 6\theta=0. \quad (14)$$

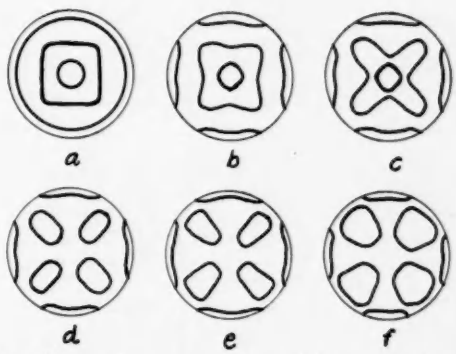


FIG. 7. All these patterns are formed from a single mathematical equation when the ratio A/B is varied.

The mathematical figures obtained from the following equations are shown in Fig. 6. They should be compared with the experimental patterns of Fig. 5, each to each

$$\left.\begin{aligned}
&4J_1\left(\frac{7.737r}{a}\right)\cos\theta+J_5\left(\frac{5.75r}{a}\right)\cos 5\theta+J_7\left(\frac{8.257r}{a}\right)\cos 7\theta=0, \quad \text{(Fig. 6b)}\\
&17J_2\left(\frac{9.16r}{a}\right)\cos(2\theta-30^\circ)+4J_4\left(\frac{4.65r}{a}\right)\cos(4\theta+30^\circ)\\
&\qquad+5J_8\left(\frac{9.39r}{a}\right)\cos(8\theta-30^\circ)=0, \quad \text{(Fig. 6c)}\\
&\left[8J_0\left(\frac{6.209r}{a}\right)-5J_6\left(\frac{6.80r}{a}\right)\cos 6\theta\right]\left[J_0\left(\frac{3.014r}{a}\right)+25J_2\left(\frac{2.292r}{a}\right)\cos 2\theta\right]=0, \quad \text{(Fig. 6d)}\\
&\left[8J_0\left(\frac{8.209r}{a}\right)+5J_7\left(\frac{8.75r}{a}\right)\cos(7\theta+13^\circ)\right]\left[J_0\frac{6.209r}{a}+50J_2\left(\frac{5.937r}{a}\right)\cos 2\theta\right.\\
&\qquad\left.+2500J_4\left(\frac{4.65r}{a}\right)\cos 4\theta\right]=0, \quad \text{(Fig. 6e)}\\
&2J_0\left(\frac{9.37}{a}r\right)+J_8\left(\frac{10.01r}{a}\right)\cos 8\theta=0. \quad \text{(Fig. 6f)}
\end{aligned}\right\}(15)$$

[5] M. D. Waller, Proc. Phys. Soc. **50**, 76 (1938).

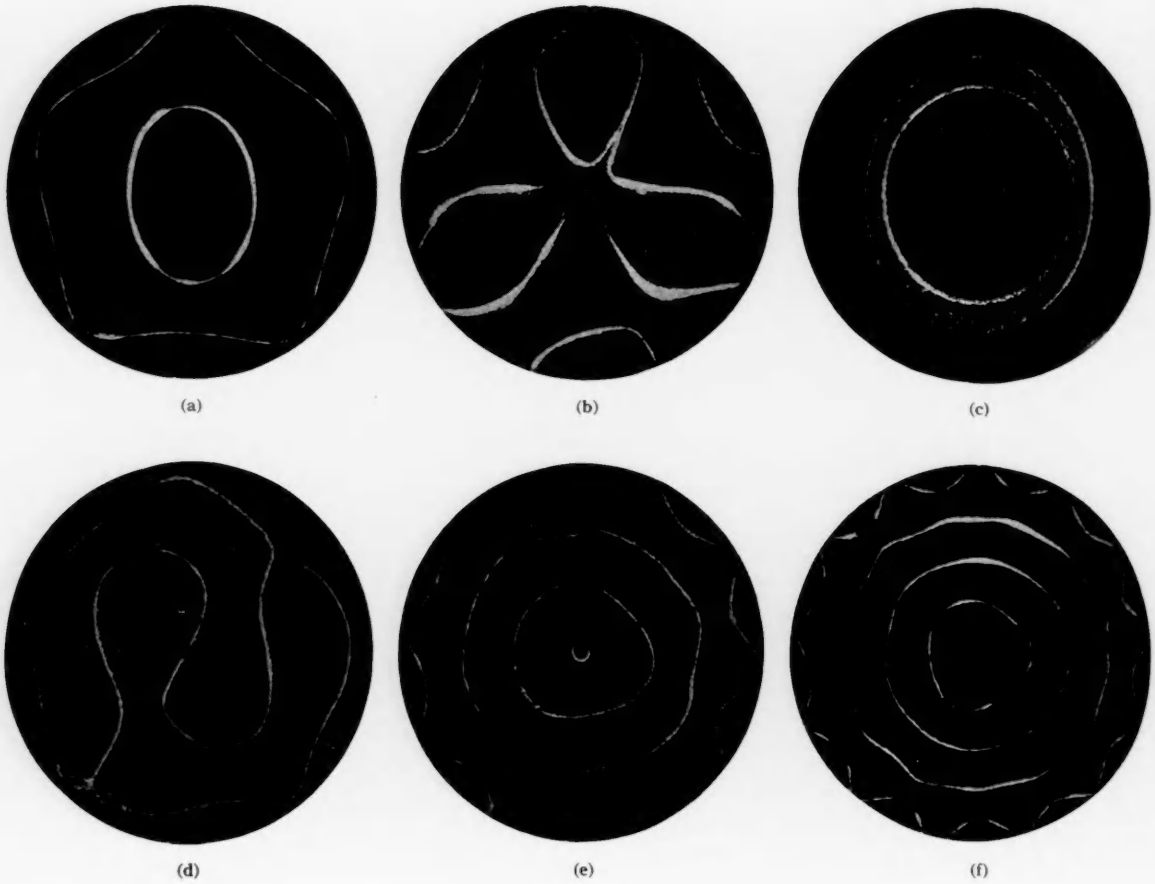


FIG. 8.

Since the first term of Eq. (1) namely $J_n(kr)$ is the expression for the displacement of a vibrating membrane, a general discussion of the membrane is also applicable to the plate. The differential equation for the membrane is

$$\frac{d^2w}{dt^2}=c^2\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}+\frac{1}{r^2}\frac{d^2w}{d\theta^2}\right). \tag{16}$$

Kirchhoff makes the assumption that $W \propto \cos(pt-\epsilon)$. By doing this, he postulates that the plate can only vibrate with a single note at any one time. Hence, his solution must give circles and diameters only. But the assumption may also be made that

$$w \propto A_1 \cos(p_1t-\epsilon_1)+A_2 \cos(p_2t-\epsilon_2)+\cdots \tag{17}$$

and

$$w=f(r,\theta)[A_1 \cos(p_1t-\epsilon_1)+A_2 \cos(p_2t-\epsilon_2)+\cdots]. \tag{18}$$

The solution of the differential equation will take the form

$$A_1J_n\left(\frac{k_1r}{a}\right)\cos(n\theta+\alpha_1)+A_2J_n\frac{k_2r}{a}\cos(m\theta+\alpha_2)+\cdots. \tag{19}$$

If the equation can be factored, the most general solution will be

$$\left[A_1 J_{n_1}\left(\frac{k_1 r}{a}\right)\cos(n_1\theta+\alpha_1)+A_2 J_{n_2}\left(k_2\frac{r}{a}\right)\cos(n_2\theta+\alpha_2)+A_3 J_{n_3}\left(k_3\frac{r}{a}\right)\cos(n_3\theta+\alpha_3)+\cdots\right]$$

$$\times\left[B_1 J_{m_1}\left(l_1\frac{r}{a}\right)\cos(m_1\theta+\beta_1)+B_2 J_{m_2}\left(l_2\frac{r}{a}\right)\cos(m_2\theta+\beta_2)\right.$$

$$\left.+B_3 J_{m_3}\left(l_3\frac{r}{a}\right)\cos(m_3\theta+\beta_3)+\cdots\right]=0. \quad (20)$$

Two or even three sets of curves will appear upon the same plate at the same time. A few of these are shown in Figs. 5 and 6.

In conclusion some diagrams will be given of the equation

$$w=AJ_0\left(\frac{9.37r}{a}\right)+BJ_4\left(8.55\frac{r}{a}\right)\cos 4\theta=0.$$

As the ratio A/B is changed, the equations (appearing in Fig. 7) are

$$\left.\begin{aligned}
3J_0\left(\frac{9.37r}{a}\right)+J_4\left(\frac{8.55r}{a}\right)\cos 4\theta=0 &\quad \text{(Fig. 7a)}\\
J_0\left(\frac{9.37r}{a}\right)+J_4\left(\frac{8.55r}{a}\right)\cos 4\theta=0 &\quad \text{(Fig. 7b)}\\
J_0\left(\frac{9.37r}{a}\right)+2J_4\left(\frac{8.55r}{a}\right)\cos 4\theta=0 &\quad \text{(Fig. 7c)}\\
J_0\left(\frac{9.37r}{a}\right)+3J_4\left(\frac{8.55r}{a}\right)\cos 4\theta=0 &\quad \text{(Fig. 7d)}\\
J_0\left(\frac{9.37r}{a}\right)+5J_4\left(\frac{8.55r}{a}\right)\cos 4\theta=0 &\quad \text{(Fig. 7e)}\\
J_0\left(\frac{9.37r}{a}\right)+10J_4\left(\frac{8.55r}{a}\right)\cos 4\theta=0. &\quad \text{(Fig. 7f)}
\end{aligned}\right\}(21)$$

The photographs of Fig. 8 show certain figures on brass plates which resemble those found by Chladni. They were produced by means of a vacuum-tube oscillator which gave out a pure sine wave.

A Physical Analysis of Distortion Produced by the Nonlinearity of the Medium

L. J. BLACK
Department of Electrical Engineering, University of California, Berkeley, California
(Received April 12, 1940)

THE distortion introduced by the nonlinearity of the air is an important consideration in the design of horn type loudspeakers for large outputs. The magnitude of the distortion can be obtained from the solution of the more exact differential wave equation. This solution has been presented in a previous number[1] of this journal. The subject has also been investigated by Goldstein and McLachlan[2] and by Rocard.[3] In this paper, the subject is treated by quantitatively following the career of a plane progressive wave. Such a treatment has the distinct advantage of giving a physical picture of the phenomenon and at the same time yielding the correct quantitative results.

The exact differential equation of wave propagation is given by Rayleigh,[4] and Lamb:[5]

$$\partial^2\xi/\partial t^2 = c^2(1+\partial\xi/\partial x)^{-(\nu+1)}\cdot\partial^2\xi/\partial x^2 = (c')^2\cdot\partial^2\xi/\partial x^2, \quad (1)$$

where $c=(\nu P_0/\rho_0)^{\frac{1}{2}}$ is the phase velocity of a wave of small amplitude, ξ is the displacement of the air particle from its equilibrium position, and ν is the ratio of the specific heats.

Equation (1) can be considered as the usual wave equation in which the phase velocity is not the same for all parts of the wave. This phase velocity, c', is:

$$c'=c(1+\partial\xi/\partial x)^{-(\nu+1)/2}=c\left(1-\frac{\nu+1}{2}\partial\xi/\partial x+\cdots\right).$$

Considering only the first two terms

$$c=c\left(1-\frac{\nu+1}{2}\partial\xi/\partial x\right)=c\left(1+\frac{\nu+1}{2}s\right), \quad (2)$$

where s is the condensation at any point in the wave.

The sound pressure in the wave is given by the expression

$$P=P_0(1+\partial\xi/\partial x)^{-\nu}-P_0=-P_0\nu\cdot\partial\xi/\partial x+\cdots = P_0\nu s+\cdots. \quad (3)$$

Making the same order of approximation as was made in obtaining Eq. (2), only the first term of Eq. (3) need be considered.

Equations (2) and (3) show that maximum condensation in a wave is at the point of maximum pressure and that this portion of the wave has the greatest phase velocity. This difference in phase velocity of various parts of the wave results in the well-known change in wave form during propagation of a wave of large amplitude.

The magnitude of the second harmonic generated as a result of the change in form of the wave, in traveling a distance x, can be obtained without the necessity of solving the exact differential equation. This alternative method avoids mathematical involvement in which the physical picture is often lost.

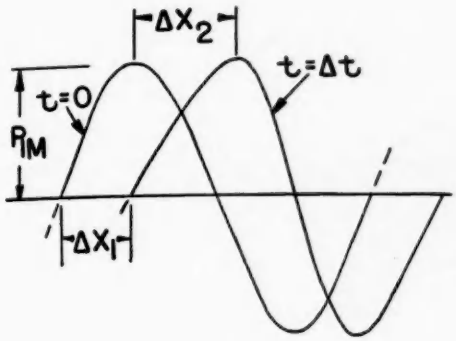
Figure 1 shows a progressive pressure wave which at $t=0$ is sinusoidal. After a time Δt, the wave has changed form because the velocity of propagation is a function of the condensation, s. The relative position of maximum pressure of the sound wave has shifted by an amount $(\Delta x_2-\Delta x_1)$.

$$\begin{aligned}\theta_1=(\Delta x_2-\Delta x_1)&=c\left(1+\frac{\nu+1}{2}s_m\right)\Delta t-c\cdot\Delta t\\ &=\tfrac{1}{2}(\nu+1)s_m c\cdot\Delta t\\ &=\tfrac{1}{2}(\nu+1)(P_{1M}/P_0\nu)\cdot\Delta x \text{ cm}\\ &=\tfrac{1}{2}(\nu+1)(P_{1M}/\rho_0c^2)2\pi(\Delta x/\lambda) \text{ radians}\\ &=\tfrac{1}{2}(\nu+1)(\omega/\rho_0c^3)P_{1M}\Delta x \quad \text{radians}, \quad (4)\end{aligned}$$

where s_m is the maximum condensation of the sound wave, ω is 2π times the frequency, and $\Delta x=c\cdot\Delta t$ is the average distance of propagation of the wave during time Δt.

To express this change in wave form in terms

[1] A. L. Thuras, R. T. Jenkins and H. T. O'Neil, J. Acous. Soc. Am. **6**, 173 (1935).
[2] S. Goldstein and N. W. McLachlan, J. Acous. Soc. Am. **6**, 275 (1935).
[3] Y. Rocard, Comptes rendus **196**, 161 (1933).
[4] Lord Rayleigh, *Theory of Sound*, Vol. 2, second edition (Macmillan, 1929), p. 31.
[5] H. Lamb, *Dynamical Theory of Sound*, second edition (Edward Arnold & Company, 1931), p. 182.



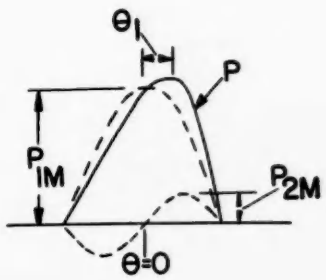


FIG. 1 (upper). Diagram showing the change in wave form during propagation of a sound pressure wave of large amplitude.

FIG. 2 (lower). Solid curve shows the wave form obtained by the addition of a fundamental and a second harmonic. The latter two are shown dotted.

of harmonic distortion, it is necessary to determine the magnitude of the second harmonic which, when added to the fundamental, will shift the position of the maximum by the angle θ_1.

In Fig. 2 is shown a wave formed by the addition of a fundamental and a second harmonic,

$$P=P_{1M}\cos\theta+P_{2M}\sin 2\theta.$$

The shift in the position of the maximum can be determined by equating $dP/d\theta=0$. For relatively small values of shift, i.e., θ_1 small:

$$\theta_1=2(P_{2M}/P_{1M}) \text{ radians}$$

or

$$P_{2M}=\tfrac{1}{2}P_{1M}\theta_1=\tfrac{1}{4}(\nu+1)(\omega/\rho_0c^3)P_{1M}^2\cdot\Delta x.$$

Expressing the result in terms of r.m.s. values and replacing Δx by x (since the distortion is a linear function of the distance of propagation):

$$P_2/P_1=\frac{\nu+1}{2\sqrt{2}}\frac{\omega}{\rho_0c^3}P_1x. \tag{5}$$

Equation (5) is, of course, the same equation that is obtained from the solution of the more exact differential equation. The usual mathematical treatment involves the same approximations[1] as are made in the analysis presented in this paper. The analysis from the physical viewpoint leads at once to the correct numerical result and at the same time builds up a picture of the processes involved. This physical picture allows a straightforward interpretation of the results. For instance, it shows clearly why the distortion is proportional to the frequency. Reference to Eqs. (4) shows that the shift expressed in cm is independent of the frequency but is directly proportional to the frequency when expressed in radians.

The Interference of Tones in the Cochlea*

ERNEST GLEN WEVER, CHARLES W. BRAY AND MERLE LAWRENCE
Princeton University, Princeton, New Jersey

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THREE types of acoustic phenomena, beats, combination tones, and masking, are generally known to result from the simultaneous stimulation of the ear by two tones. The present paper describes a fourth type, that of tonal interference.

Interference appears in the electrical activity of the cochlea as a reduction in the magnitude of response to a given tone when a second tone is simultaneously sounded. In this respect it resembles masking as shown in subjective observations, yet the two phenomena are distinct. Interference occurs between tones of any frequency, whereas masking is limited to frequencies in the region of the masking tone and its harmonics. We have not observed in the cochlear response a pattern of interaction that resembles masking, and consider this phenomenon to arise beyond the cochlea, probably in the early synapses of the acoustic nervous system.

Interference was discovered in the cochlear response by Covell and Black.[1] They observed it for various tones and recognized its independence of masking, but did not enter into a special study of its character. The present experiments employ the cochlear response to study the phenomenon in its relations to the stimuli and to other auditory processes.

METHOD AND PROCEDURE

The experiments were performed upon guinea pigs and rabbits under urethane anesthesia. The stimuli were pure tones from a loudspeaker actuated by currents from two audiofrequency channels. Each channel consisted of an oscillator, attenuators, filters, and other controls. In most of the experiments the sounds from the loudspeaker were conducted through a tube to the meatus of the animal's ear. In a few, where precise measurements of frequency sensitivity were desired, the meatus was placed at a standard distance of 2.5 cm from an open tube connected with the speaker. Under our conditions the open-tube arrangement permitted a more accurate calibration of stimulus intensity.

The cochlear potentials were picked up with a silver foil electrode on the membrane of the round window, and after amplification were studied with a wave analyzer (General Radio Type 736-A), and at times with a cathode-ray oscillograph. The wave analyzer is particularly useful for the selective measurement of any desired component of the response.

Tests were made to show the absence of intensity distortion in the stimulating and recording apparatus. A crystal microphone was used to pick up the sounds, and its output was passed through the amplifier to the wave analyzer. It was found that the reading obtained for one tone was unaltered by the introduction of a second tone at any intensity within the range employed in the experiments, provided that the two tones were at least 15~ apart. The latter condition is due to the limit of selectivity of the wave analyzer.

In certain experiments, mentioned below, a crystal vibrator was substituted for the loudspeaker in one or both channels for mechanical stimulation either through the skull or by direct actuation of the stapes.[2] Tests were carried out with this instrument also to show the absence of distortion in its operation, for all intensities employed in the experiments.

Results were obtained on 20 guinea pig ears and 7 rabbit ears.

RELATION TO STIMULUS INTENSITY

If during observation of the response to a given tone a second or interfering tone is introduced and progressively raised in intensity, the response

* From the Princeton Psychological Laboratory. This investigation was aided by a grant from the Rockefeller Foundation.

[1] W. P. Covell and L. J. Black, "The cochlear response as an index to hearing," Am. J. Physiol. **116**, 524–530 (1936).

[2] For details of technique, see E. G. Wever, C. W. Bray, and M. Lawrence, "The locus of distortion in the ear," J. Acous. Soc. Am. **11**, 427–433 (1940).

at first is unaltered, and then when a suitable intensity is reached it suffers a reduction in magnitude. Numerous experiments were carried out to study this impairment of the response and to determine its dependence upon the intensity of the two stimuli.

Figure 1 shows results obtained on a guinea pig ear by the introduction of a 3000~ tone during measurements of the responses to a 1000~ tone. The ordinate shows the magnitude of response for the 1000~ tone in microvolts at the round window, and the abscissa indicates the intensity of the 3000~ interfering tone in dynes per sq. cm. The 1000~ tone was adjusted to give a normal response, in the absence of the interfering tone, of about 10μv, and its intensity was maintained at this value, 0.04 dyne per sq. cm.

At an intensity of about 0.05 dyne per sq. cm the effect of the interfering tone begins to be noticeable. The curve declines slowly at first, and then rather rapidly, until at the highest intensities it shows a tendency to level off.

The further course of the interference curve could not be ascertained on account of the risk of damage to the ear. Even for the higher intensities here employed there were indications of serious stress. During the application of the interfering stimulus the responses were somewhat unstable, and upon its removal they were temporarily depressed. The higher the intensity of the interfering tone the more marked the depression of the response and the longer the time required for a return to the base level. Moreover, if the intensity was carried too far, an impairment was caused from which there was no recovery. In all these experiments, therefore, the interfering tone was applied only long enough to obtain a reading, and was followed by a normal reading to make certain that no injury had resulted.

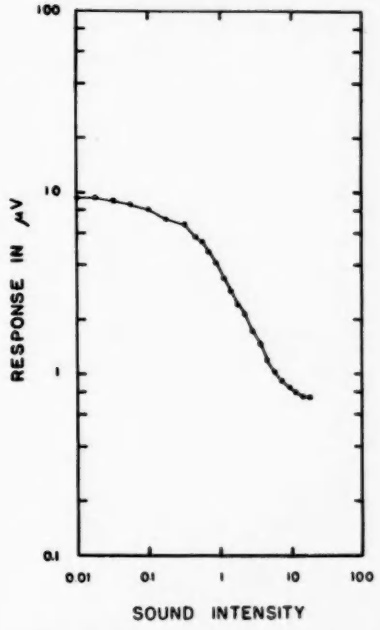

FIG. 1. Responses to a 1000~ tone (ordinate) as measured in the presence of a 3000~ tone (abscissa). Guinea pig ear, No. 280 R.

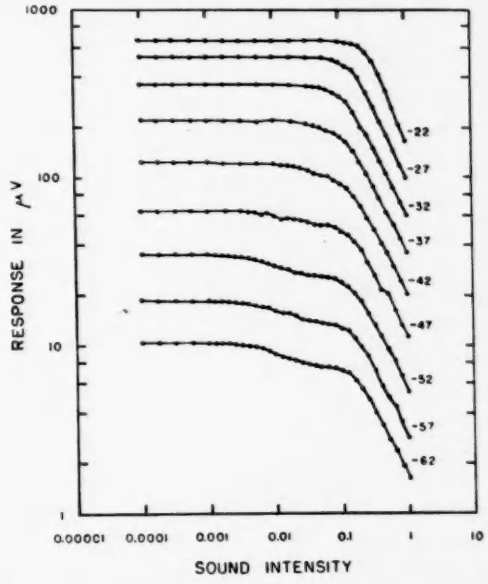

FIG. 2. Responses to a 1015~ tone (ordinate) as measured in the presence of a 3000~ tone (abscissa). The parameter is the intensity of the 1015~ tone. Rabbit ear, No. 326 R.

Figure 2 shows more extensive results for a rabbit ear. The interfering tone was 3000~ as before, but the tone acted upon was set at 1015~ to avoid any possible complication through the harmonic relation of the two stimuli (a precaution which the results prove to be unnecessary).

The coordinates for this figure are the same as for the previous one, and the parameter is the intensity of the 1015~ tone. This tone was first adjusted to an intensity that gave a normal response of 10 μv, as shown by the lowermost curve, and then for the remaining curves was raised by steps of 5 db. The numbers at the ends

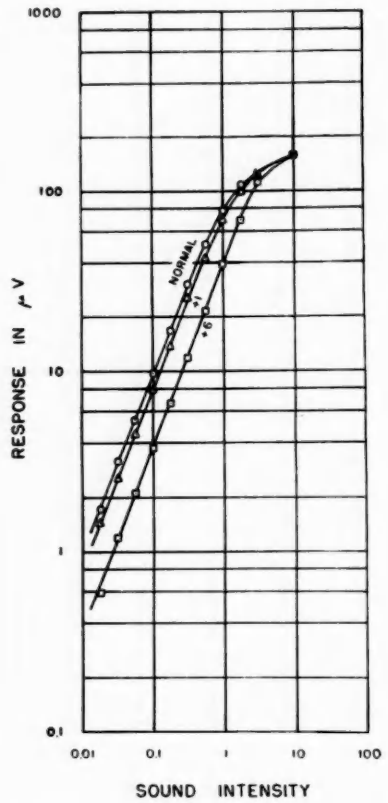

FIG. 3. Intensity functions for a tone of 10,015 ~ under normal conditions and in the presence of a 3000 ~ interfering tone at each of two intensities, indicated as +1 and +6 db. Guinea pig ear, No. 319 R.

of the curves show the intensities in db with respect to 1 dyne per sq. cm.

The curves of this figure all have the same general form as in the preceding figure, except that extreme intensities were not used for the interfering tone, and no tendency to level off is disclosed. Several features appear as a function of the intensity of the tone acted upon.

The lowermost curves contain a preliminary dip, which begins at a moderate intensity of the interfering tone and then goes over into a rapid decline like that shown by the other curves. This preliminary effect was irregular in appearance; it was not present, for example, in the curve shown in Fig. 1. It is possible that it is due to a partial contraction of the tympanic muscles, and thus is extraneous to the phenomenon under consideration.

A prominent feature of the curves is the intensity of the interfering tone at which the rapid flexure appears. This intensity is nearly the same for all the lower curves, but it moves rather rapidly upward for the three highest curves. Also, the flexure becomes considerably sharper for the uppermost curves. After the flexure, the downward course of the curves is closely similar for all except the three highest curves, for which it is somewhat steeper. A further significant feature is the separation of the curves and their general course with respect to one another; but this feature will appear more clearly in the next figure where a different mode of presentation is employed.

Figure 3 shows the effect of a 3000~ tone upon a tone of 10,015~ in a guinea pig ear, with a form of plotting that brings out the relation of interference to the intensity of the tone acted upon. Here the abscissa shows the intensity of the tone acted upon, while the parameter is the intensity of the interfering tone. The ordinate gives the response to the tone acted upon, as before.

The curve marked "normal" is the intensity function for the 10,015~ tone in isolation; the other two curves show the responses obtained to this tone in the presence of the interfering tone at intensities of +1 and +6 db with respect to 1 dyne per sq. cm, or 1.1 and 2.0 dynes per sq. cm, respectively.

Except at the highest intensities the interference curves run nearly parallel to the normal curve. Interference thus is shown, within the limit indicated, to be almost a constant fraction of the normal response; for the +1 db curve this fraction is 0.83, and for the +6 db curve it is 0.40. These two fractions expressed in db amount to 1.6 and 8 db, respectively.

At their upper ends, where they depart from a straight form on double-logarithmic coordinates, the curves reflect more complex relations. Here all the functions show "overloading" as a rapid departure from their simple course.

Though all the functions bend over, the interference curves present notable differences. In the region of intensity where the normal curve is beginning to bend, the interference curves still remain straight; it appears that in this region the decrease of response brought about by interference has delayed the onset of overloading. When the intensity is carried higher, overloading sets in for the interference curves also, and proceeds

at a rapid rate. At the highest intensity employed, the curves meet at a common point.

In the region where the curves are bending the vertical separation is diminished, and finally is reduced to zero at the last point shown. These results might be taken to mean that interference grows less and finally disappears at high intensities of the tone acted upon, but in view of its constancy through the lower course of intensity variations it seems more reasonable to suppose that here its action has been obscured by overloading.

The results suggest that overloading is a function of the magnitude of a local process, and as interference reduces this magnitude it protects the responses from overloading. These considerations carry the implication that overloading takes place at a stage later than the process of interference in the action of the cochlea.

Relations to Frequency

Interference occurs between any pair of tones, whatever their frequencies. It is found for tones but a few cycles apart, like 1000~ and 1015~, and for tones at the ends of the frequency scale, like 100~ and 10,000~. This does not mean, however, that frequency is wholly irrelevant. As will be shown, the degree of interference for tones of given intensity varies as a function of frequency.

In the study of frequency relationships, a number of experimental procedures are possible, since there are several variables. These variables include the frequency, intensity, and response level of each tone; and the observations may consist either of the amount of interference obtained for some constant change in one of the stimuli or, conversely, of the amount of change of stimulus necessary to give a standard degree of interference. Finally, the changes may be considered either absolutely or relatively. We have explored nearly all combinations of these procedures, and report here the ones that have yielded the most meaningful results.

Figure 4 shows curves obtained on a guinea pig ear by three procedures, together with a sensitivity curve for the cochlear response. The three interference curves were obtained with a tone of 1015~, adjusted to give a normal response of 10 μv, and then acted upon by each of 11 different frequencies over the range from 100~ to 10,000~ as shown along the abscissa. The ordinate is a decibel scale for which the zero or reference line has two meanings, as indicated below.

For the curve marked "1 dyne" the interfering tones were all adjusted to an absolute intensity of 1 dyne per sq. cm, and the amount of interference was observed for each. For this curve, therefore, the zero of the decibel scale refers to the normal response of 10 μv, and points below show reductions from this level in db. It will be seen that every interfering tone impairs the response, but not in equal degree. The curve contains two minima, one, which is slight, in the region of 700–2000~, and another, more prominent, in the region of 7000~. Minima on this curve indicate regions of greatest interference.

The curve marked "1 db loss" was obtained by adjusting the intensity of each interfering tone until a reduction of response of 1 db was obtained. The intensity necessary to produce this reduction is indicated on the decibel scale with reference to 1 dyne per sq. cm; points below the zero line represent db below 1 dyne per sq. cm. Lower values on this curve thus have the same significance as on the "1 dyne" curve, in that

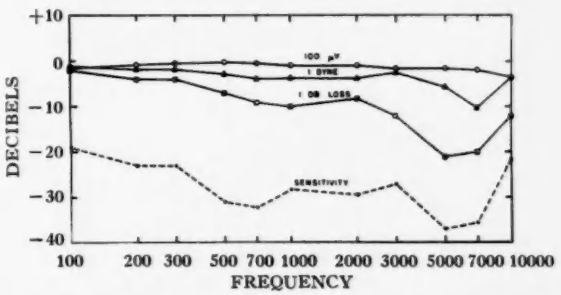

Fig. 4. Interference and sensitivity curves for a guinea pig ear, No. 323 R.

they indicate relatively greater susceptibility of the 1015~ tone to interference. The curve has much the same form as the "1 dyne" curve, but the variations are more pronounced, and the minima are located a little below. A great many other curves like these were obtained in this and other animals, and in general the "1 dyne" and "1 db loss" curves showed similar variations with frequency.

The two curves just mentioned are to be com-

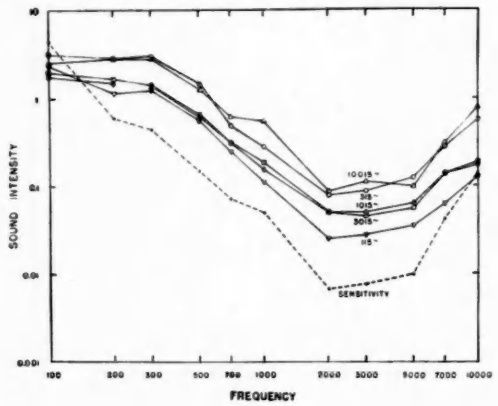

FIG. 5. Interference and sensitivity curves for a rabbit ear, No. 325 R.

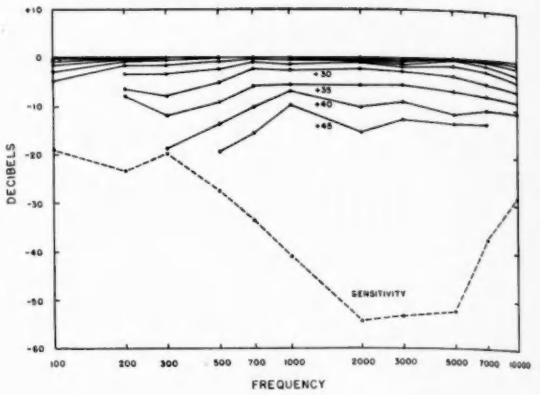

FIG. 6. Interference and sensitivity curves for a rabbit ear, No. 325 L.

pared with the "sensitivity" curve below. The latter curve shows the stimulus intensity necessary for each of the 11 tones themselves to produce a normal response of 10 μv. For this curve the decibel scale is referred to 1 dyne per sq. cm, and thus lower points indicate smaller intensities required for the standard response and hence greater sensitivity. Two regions of relatively high sensitivity are shown, one around 500–700~ and another, which is more marked, around 5000–7000~.

There is a considerable resemblance between the sensitivity curve and both the "1 dyne" and "1 db loss" curves. The resemblance is not perfect: the variations in the interference curves are smaller, and there is noticeable disparity in the slopes at the low and high ends of the frequency scale; yet the correspondence is sufficient to suggest an intimate relation between the processes involved.

If interference were a direct function of sensitivity it should be the same for tones which are adjusted for equal response. The upper curve of the figure, marked "100 μv" shows results where this adjustment was made. The interfering tones were first presented alone, and adjusted in intensity to give a standard response of 100 μv. They were then presented as interfering tones at the intensities thus determined, and their effects upon a tone of 1015~ were ascertained. The 1015~ tone itself was adjusted to give a normal response of 10 μv, and the decibel scale shows the observed response with reference to this normal value.

It is plain that this curve is not perfectly flat, as it should be if interference were a simple function of sensitivity. The uniformity is considerable, however, and the two depressions shown by the other interference curves are now absent. The curve falls away noticeably at the two ends of the scale, and thus it shows in these regions the intervention of some condition which is different for the interference and sensitivity functions.

Further investigation of the relationships just described was carried out on the rabbit because this animal shows a different kind of sensitivity curve, with a considerably greater range of variation. These results will be presented in a manner to show also the effects of varying the frequency of the tone acted upon.

Figure 5 represents observations on each of 5 tones, 115~, 315~, 1015~, 3015~, and 10,015~, when acted upon by 11 different tones. The curves are all "1 db loss" curves: they show the sound intensity necessary for each of 11 different tones to produce a decrease of 1 db in the tone acted upon. The tone acted upon was presented at an intensity that gave a normal response of 10 μv. The sixth curve in the figure is a sensitivity curve, and shows the intensity necessary for each of the 11 tones when presented alone to give a response of 10 μv.

It is evident that the 5 interference curves resemble one another in form, and that all follow much the same course as the sensitivity curve. Just as was found true for the guinea pig, the chief departures from the sensitivity curve are at

the two ends of the scale. In this animal the differences at the low frequency end are particularly striking.

In Fig. 6 appear results on another rabbit ear, which show the effect of variations in the stimulus level of the interfering tones.

The tone acted upon had a frequency of 1015~, and was presented at an intensity that gave a normal response of 10 μv; points below the zero line thus represent db of interference.

The interfering tones were first adjusted to produce a response of 10 μv when presented alone, and then were raised by successive steps of 5 db up to the maximum that was considered safe to deliver to the ear. This maximum varied for different frequencies, and was generally less at the two ends of the scale, but for most tones it extended as high as 45 db above the intensity that gave the standard level of 10 μv. The numbers on the curves show the intensity used, in db above the base as defined. A sensitivity curve is included as in the two previous figures. The decibel scale for this curve is referred to 1 dyne per sq. cm, and the points given indicate the intensity required at each frequency to produce a standard response of 10 μv.

Except at the ends of the scale, the intensities below 20 db produced only slight amounts of interference. The curves therefore are crowded near the zero line, and in the middle portion of the graph many of them are indistinguishable from that line. The curves that are not marked can be identified by counting in 5-db steps from the labeled curves.

As was found for the guinea pig, curves of interference like these, in which an adjustment has been made for equivalence of response, no longer resemble the sensitivity curve. Through the middle of the frequency range, and at moderate levels of intensity, the curves are fairly flat, and show within limits a considerable degree of correspondence between interference and sensitivity. There is a falling away of the curves toward both ends of the scale, and especially at the lower end.

At the highest intensity levels the curves are somewhat irregular. These irregularities evidently are related to the fact that at such levels equal increments of stimulus no longer produce equal amounts of response,[3] but the present data are not extensive enough to show the relations in detail. Moreover, the behavior of the curves at the lowest frequencies could not be ascertained due to the risk of injury.

The evidence of this section is in agreement in showing a close relation between cochlear sensitivity and interference throughout the frequency scale. The consistency of the results is the more striking because it obtains for two different species of animal, with different forms of sensitivity.

Relation to the Transformation Principle

The question is raised whether interference can be accounted for simply on the basis of the transformation principle. According to this principle, sounds applied to the ear in sufficient strength are subjected to distortion so that a portion of their energy is transformed into other frequencies. A pure tone applied to the ear thus sets up a series of overtones, and two such tones produce not only overtones but combination tones also.

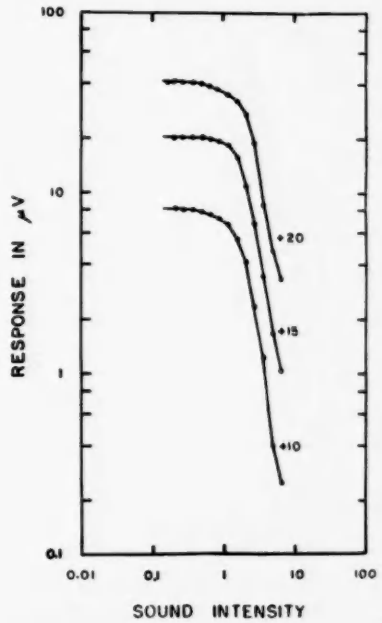

FIG. 7. Effect of interference upon the second harmonic. Responses shown are the 5600~ component for a 2800~ stimulus, interfered upon by a 4000~ tone. Guinea pig ear, No. 302 R.

[3] Cf. E. G. Wever and C. W. Bray, "The nature of acoustic response: the relation between sound intensity and the magnitude of responses in the cochlea," J. Exper. Psychol. **19**, 129–143 (1936).

TABLE I.

INTENSITY OF 2800~ TONE	INTENSITY OF 1000~ TONE −5	+5	+15
0	88.8	124.6	220.2
	86.0	*112.8*	*210.5*
	96.6	**90.4**	**95.7**
+10	147.1	169.6	238.3
	145.0	*150.9*	*172.6*
	98.5	**88.9**	**72.7**
+20	200.0	201.7	240.9
	196.8	*188.2*	*184.4*
	98.5	**93.2**	**76.4**

Previous work[4] has shown that the ear actually operates according to this principle: that single pure tones give rise to a complex response in the cochlea which when analyzed yields a Fourier series; and two pure tones give in addition to their series of overtones all the combination tones as defined by the formula $m\,h \pm n\,l$, where h, l are the frequencies of the higher and lower stimuli, and m, n are any simple integers.

In the process of transformation itself no stimulus energy is lost; that which is withdrawn from the fundamentals enters into the overtones and combination tones, and the total sum is constant. (It is assumed that frictional losses are not significantly greater under conditions of distortion than under other circumstances.) To account for interference on the transformation principle it is necessary merely to suppose that the decrease observed in the fundamental component of a response when a second stimulus is applied is due to the diversion of energy into other components.

We may first consider this hypothesis on a relatively simple level, by testing whether during interference the decrease of response observed in the fundamental component of a stimulus is compensated for by a rise in its own harmonics.

Figure 7 shows the effect of an interfering tone of 4000~ upon a component of 5600~, obtained as the second harmonic of a 2800~ stimulus. The intensity of the interfering tone is shown on the abscissa, and the magnitude of the 5600~ component is shown on the ordinate. The numbers on the curves indicate in db the intensity of the 2800~ stimulus tone relative to 1 dyne per sq. cm.

It is apparent that interference operates not only on the fundamental component of a response, but also on the second harmonic. Similar effects have been observed on the other harmonics. Diversion of response to the overtones of a tone acted upon by an interfering stimulus thus does not occur; on the contrary the overtones are reduced.

A study was made to ascertain whether the interfering tone acts upon the harmonic components directly, or whether it brings about their reduction only because the fundamental itself has been impaired. It is well known that the amount of harmonic distortion is a function of stimulus intensity, and it is conceivable that interference simply reduces distortion by reducing the effective intensity.

Observations were made on the second and third harmonics. These were measured when a given tone was presented alone and again in the presence of an interfering tone, with adjustment of the intensity of the tone acted upon so that the same value of fundamental component was obtained in each instance. If the supposition were correct that the amount of distortion is due only to the effective stimulus as indicated by the fundamental response, then these two conditions should give the same amount of harmonics. The facts are otherwise: in the interference situation the harmonics are notably less.

The following is an example of these results. A tone of 1000~ when presented alone at an intensity of 0.10 dyne per sq. cm gave a response in which the fundamental component amounted to 168 μv and the 2000~ component was 10.5 μv. The same tone when given at 0.28 dyne per sq. cm in the presence of a 2800~ interfering tone gave a response in which the fundamental component again was 168 μv, but the 2000~ component was only 2.8 μv. The difference in the second harmonic, which amounts to 11.5 db, represents interference upon that harmonic itself. In this same case the third harmonic was reduced from 6.4 μv to 3.8 μv, and thus suffered interference to the extent of 4.5 db.

For a complete determination of the pattern of response during interference it is necessary to make measurements on many components, and

[4] E. G. Wever and C. W. Bray, "Distortion in the ear as shown by the electrical responses of the cochlea," J. Acous. Soc. Am. **9**, 227–233 (1938); E. G. Wever, C. W. Bray, and M. Lawrence, "A quantitative study of combination tones," J. Exper. Psychol. **27**, (1940) (in press).

to include not only the harmonic series of each tone but also the combination tones. As both the harmonic series of a single tone and the combination tone series of a pair of tones are rapidly convergent it is possible to limit the observations to a practicable number of components.

Measurements were made upon the fundamental and first 4 overtones in the response to each of two stimuli when these were presented alone, and also upon these components and 12 among the earlier orders of combination tones when the stimuli were presented together. The total response for each set of measurements was obtained as the square root of the sum of the squares of the individual measurements. Measurements were carried out on 4 guinea pig ears, in each case with several different intensities of the stimulus tones.

Before considering the results themselves it is necessary to point out that the crucial question in this situation has to do with energy distribution, whereas our measurements are of cochlear potentials. Potential measurements show energy relationships only if the effective impedances are equal, or can be taken into account. In the present instance the impedances desired are those that obtain at the locus of the transformation process, and operate from that point to the place at which the potentials are recorded.

No method is known for determining these impedances directly. They are of course reflected in the sensitivity function for the cochlear response, but the difficulty is that this function reflects also the mechanical characteristics of the more peripheral portions of the ear. This difficulty disappears, however, if it can be assumed that there is close coupling between the peripheral structures and the process in the cochlea through which distortion arises, for then the impedances will follow the sensitivity function closely. This assumption is reasonable on general grounds, and accords with the high degree of resemblance found between the sensitivity curve and interference as a function of frequency.

Results obtained on one ear are shown for stimuli of 1000~ and 2800~, presented each at 3 different intensities as indicated and thus making 9 sets of measurements in all. The results are given in two forms, before and after correction for sensitivity variations. Table I shows the root-mean-square totals for the components as measured without correction. Table II shows these totals after correction of the value of each component on the assumption that impedance variations follow the form of the sensitivity curve of the ear studied.

Each division of the tables presents a single stimulus arrangement, and contains 3 numbers. The first number, in roman type, shows the total for the normal condition, when the two stimuli were presented separately. The second number, in italics, shows the total for the interference situation, when the two were given in combination. The third number, in bold face, expresses the percentage that the second number is to the first, and thus indicates the degree of agreement between the two conditions.

It is evident that the agreement is closer in Table II, after the corrections described, though the differences are small. The percentages do not in any case reach 100, but for the more moderate intensities of stimulation they are within a few percent of that value. Somewhat closer agreement no doubt would have been found if the measurements had included a larger number of components.

When both stimuli are strong the correspondence between the two sets of measurements is significantly less, and it is certain that the inclusion of more components would not have brought the percentages appreciably closer to the 100 value.

It appears from these results that under suitable conditions the loss in the fundamental component of response that is occasioned by the presence of a second stimulus is in large part at

TABLE II.

INTENSITY OF 2800~ TONE	INTENSITY OF 1000~ TONE −5	+5	+15
0	126.7 *123.8* **97.5**	142.4 *128.9* **91.4**	231.8 *223.8* **96.5**
+10	218.5 *217.2* **99.2**	236.1 *221.9* **94.1**	290.7 *230.9* **79.5**
+20	303.8 *299.7* **98.7**	298.0 *287.0* **96.3**	312.4 *275.5* **88.5**

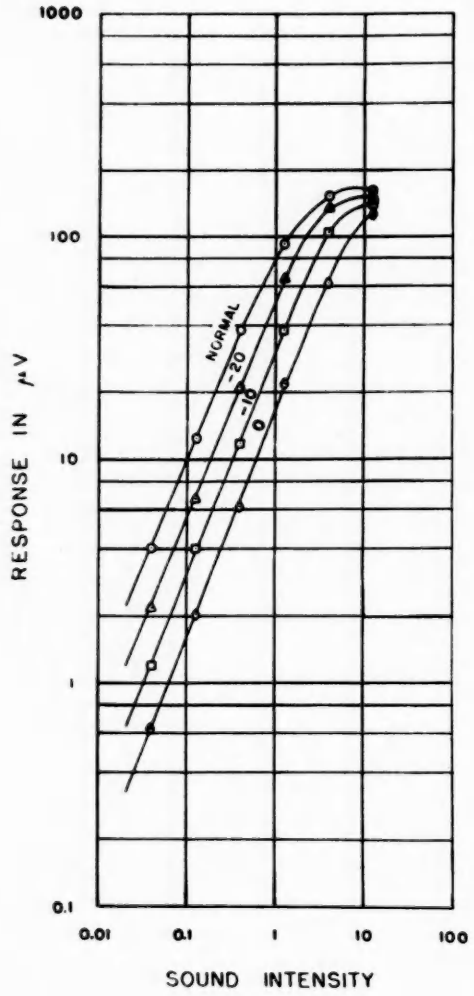

FIG. 8. Intensity functions for a 3000~ tone, measured in the presence of a 6100~ tone applied by bone conduction. Guinea pig ear, No. 279 R.

least accounted for as a diversion to other components, the combination tones. An essential relationship between transformation and interference has thereby been revealed.

When both stimuli are strong an actual loss of response occurs. However, this fact does not necessarily signify that the relation mentioned between transformation and interference is limited to the moderate intensities of stimulation, but rather that at extreme intensities overloading enters to obscure this relationship. It is to be expected that overloading as considered for all components as a whole will be relatively greater for two strong tones applied together than for these tones in isolation.

THE LOCUS OF INTERFERENCE

Previous studies of overtones and combination tones have shown that these phenomena do not appear in appreciable amounts in the more peripheral portions of the auditory apparatus, but arise beyond the stapes, most probably in the action of the hair cells of the organ of Corti.[5] As the evidence just presented indicates that the interference process is closely allied to the transformation by which overtones and combination tones are formed, it is readily inferred that interference likewise arises in the cochlea. Direct evidence for this view will now be presented.

Figure 8 shows interference functions obtained with a tone of 3000~ conducted to the ear in the usual manner, and an interfering tone of 6100~ applied by bone conduction. For the bone-conduction stimulus one of the audiofrequency channels was connected with a crystal vibrator whose plunger was applied to the occipital bone.

The intensity of the air-conduction tone is shown on the abscissa, and the resulting response on the ordinate; the parameter is the intensity of the interfering bone-conduction tone. As the vibrator used for the bone-conduction stimulus was not calibrated, the intensity was not known exactly. The values given are the intensities for air-conduction stimuli that yield equivalent responses; the reference level is 1 dyne per sq. cm as usual. The upper curve, marked "normal," is the function in the absence of the interfering tone, and the other curves are interference curves obtained at the levels of intensity indicated.

These results resemble those obtained under usual conditions, when both tones are aerially conducted. No significant variation has been introduced by the new path of conduction.

As it is often argued, and plausibly, that the middle ear may be in operation even for bone conduction, the above evidence is not final. Further experiments were performed which excluded beyond dispute any action of parts peripheral to the stapes. In these experiments the ossicular chain was broken at the incudostapedial joint, and the point of a needle attached to a crystal vibrator was applied to the head of the stapes.

[5] E. G. Wever, C. W. Bray, and M. Lawrence, "The locus of distortion in the ear," J. Acous. Soc. Am. **11**, 427–433 (1940); "The origin of combination tones," J. Exper. Psychol. **27**, 217–226 (1940).

Both the interfering stimulus and the one whose responses were measured were produced by the vibrator; suitable tests showed the absence of distortion in the apparatus under these conditions.

Observations made on interference by stapes stimulation are in agreement with those already described. Figure 9 presents some of these results. The interfering tone had a frequency of 1500~, and was varied in intensity as indicated on the abscissa. The tone acted upon had a frequency of 1000~, and was presented at 5 different intensities as given at the ends of the curves. The ordinate shows the magnitude of response for the 1000~ tone; the dashed lines indicate the probable course of the curves up to the point where interference measurements began. The values of intensity shown were approximated as in the foregoing experiment.

These results are similar to those obtained when the ear was intact, and show beyond question that structures peripheral to the stapes are not responsible for interference.

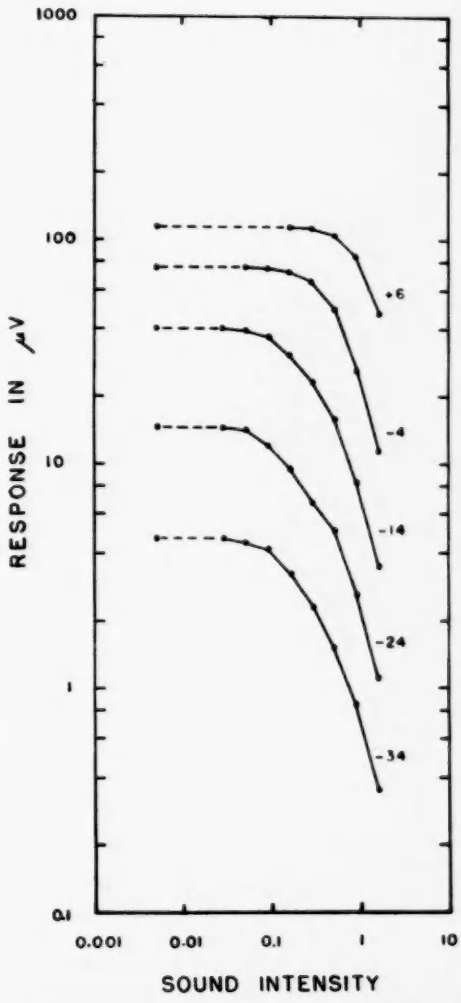

FIG. 9. Interference in the absence of peripheral parts of the ear. Responses are to a 1000~ tone in the presence of a 1500~ tone, with both stimuli applied to the stapes. Guinea pig ear, No. 290 R.

DISCUSSION

In the course of this investigation we have gained new insight into the character and order of events in the cochlea. From the entrance of sound waves into the meatus to the generation of the cochlear potentials runs a series of complex processes, mechanical and electromechanical, which may be viewed in terms of the functional relations between the recorded potentials and the variable dimensions of the stimuli. Thereby an analysis of cochlear activity into particular processes is effected. It should be emphasized that such an analysis is independent of our ability to distinguish either anatomically or physiologically the mechanisms which the processes represent. We offer suggestions as to possible mechanisms because they give concreteness to our conceptions, and aid in the projection of further investigations.

We have distinguished six processes, as follows: (1) peripheral conduction, (2) transformation, (3) interference, (4) overloading, (5) injury from overstimulation, and (6) generation of cochlear potentials. These will now be considered in their relations to one another and to the intensity and sensitivity functions of the cochlear response.

The peripheral conductive processes comprise a whole series of closely linked mechanical conditions. They include the processes of the middle ear and such mechanical properties of the inner ear, in the fluid pathways and in the sensory receptor cells themselves, as determine the effective sensory stimulus. Experimental procedures have shown that the earlier of these processes, those before the stapes, are incidental to the phenomena that are our present concern, for transformation, interference, overloading, and injury through overstimulation all continue to occur when these processes are excluded. The conductive processes generally, however, includ-

ing those of the inner ear, are important in their effect upon frequency sensitivity; the evidence indicates that in the determination of the form of the sensitivity pattern these are the chief, though not the exclusive, conditions. They are important also in that they determine, for stimuli of given frequency and intensity, the particular locus and spread of action over the basilar membrane.

Transformation, the process by which a portion of the energy in a stimulus is thrown into overtones and combination tones, takes place almost certainly in the cochlea, and most probably in the hair cells of the organ of Corti. The evidence indicates that it is one of the earliest of cochlear processes, and a likely explanation is that it arises in the deformation of the hair cells by the conducted stimulus. For slight deformations we may expect these cells to obey Hooke's law, but for extreme deformations they almost inevitably must fail to do so. The result will be the formation of overtones and combination tones, in the manner observed.

In acoustical systems of all kinds, wave form distortion and intensity distortion are generally found together, and often arise from a common cause. In the present situation, interference has been observed to be closely allied to the process of transformation, and one may suppose that here also the two depend upon a common operation. It is likely that as deformation of the hair cells becomes extreme the effectiveness of the movements in further excitation grows progressively less, and finally approaches a limit. Interference occurs between two tones because they operate upon hair cells in common, and the presence of the second tone affects the action that can occur in response to the first. The stronger the interfering tone the greater is its effect, as the other tone is caused to work under progressively less favorable conditions. Tones of only moderate strength, of course, will not interfere, but will superimpose their effects, since their operations are restricted to the range where the hair cell responds in linear fashion.

The conditions that have been assumed as the basis of transformation and interference would give an intensity curve that ran a simple course at low levels and then approached a limit at high levels. These characteristics ought to be represented in the function obtained for the cochlear response when stimulus intensity is varied. This function, as has repeatedly been shown, is indeed simple for low and moderate intensities, and then it bends, at first gradually and then more quickly until it passes through a maximum. As the stimulus intensity is carried beyond the maximum point, the response decreases rapidly, becomes unstable, and finally suffers an impairment from which there is no immediate recovery.

In descriptions of the intensity function for the cochlear response, the term overloading has been applied to the entire upper part of the function where it departs from simple form. From considerations that derive from the present investigation we consider the form of the function at the higher levels to be due to two influences. One, already referred to as transformation and interference, gives the early bending of the function and its approach to a limit. The second, which we regard as overloading proper, is a decline in effectiveness for response. It begins to operate when the intensity becomes extreme and progresses rapidly until eventually it carries over to actual injury. This effect probably begins a little before the maximum of the response is attained, increasing the bending of the function, and giving the curve a maximum in place of a limiting level.

The evidence indicates that the process of overloading is secondary to the processes of transformation and interference. This evidence comes from two types of observation, first, that in the presence of an interfering tone the onset of overloading is delayed, and, second, that when overloading does appear for two strong tones it is relatively greater, considered for all the components, than for the same tones presented singly. In the first case the presence of interference seems to act, within limits, as a protection against overloading; and in the second case the overloading is so great that it overlies and obscures interference.

Overloading probably appears somewhere between the deformation of the hair cells and the action by which the cochlear potentials arise, or as a phase of the latter action. It seems to be some change that impairs the capacity of the cells to generate the electrical potentials, some change perhaps in their electrochemical state.

Ordinarily the change is momentary, though at the extreme intensities there is much instability, and at times general variations of response occur. The injury that results from overstimulation is probably this same change carried so far as to be irreversible.

Sensitivity as shown by the cochlear potentials is plainly a complex function, and reflects the properties of the whole series of processes from the entrance of the stimulus to the recording of the potentials at the round window. The degree of resemblance between the interference and sensitivity functions indicates that the chief factors in the determination of sensitivity enter early, and evidently are the mechanical characteristics of the conductive system up to and including the hair cells. Other factors enter beyond the point where interference occurs; and for both the species studied these factors operate primarily to reduce the effectiveness of frequencies at the two ends of the scale.

Conclusions

A new phenomenon, which is called tonal interference, is revealed by the electrical responses of the cochlea as a reduction of the response to one tone due to the simultaneous presentation of another.

This phenomenon is distinct from masking, as shown particularly in two features: (a) it occurs between tones of any frequency, while masking is limited to tones in the region of the masking tone and its harmonics; and (b) it is shown in the cochlear responses and hence is peripheral in origin, while masking is not revealed in these responses and evidently is a consequence of nervous action.

Beyond the points just mentioned, the phenomenon manifests distinctive relations to the stimuli and to other auditory processes, as follows:

1. As the intensity of the interfering tone is increased, the interference effect sets in rather abruptly, and then proceeds at a rapid rate until at a very high intensity it tends to level off.

2. Interference is practically independent of the intensity of the tone acted upon, except at higher levels where it becomes obscured by overloading.

3. Interference occurs between tones of any region of frequency and any frequency ratio.

4. The amount of interference varies with frequency in a manner that suggests that it depends upon the effective intensity at the locus of the interference process. The frequency curve resembles the sensitivity curve throughout the middle range, but departs somewhat for low and high tones.

5. An intimate relation is found between interference and the process of transformation by which overtones and combination tones are produced in the cochlea. It is shown that in the interference situation, with stimuli that are not excessive, there is little or no loss of energy, but rather a diversion to new components. That which is lost in the fundamental and overtones is accounted for in the rise of combination tones.

6. Interference occurs independently of the more peripheral portions of the conduction apparatus of the ear; its most probable locus is the hair cells of the organ of Corti.

7. The results indicate that interference occurs relatively early in the series of operations from the entrance of sound waves into the ear to the generation of the cochlear potentials. It is closely connected with the process of transformation, and may be simply a phase of that same process.

8. Overloading evidently follows the process of interference. It is found that (a) when the fundamental component is observed, the presence of interference delays the onset of overloading, while (b) when the total response is considered, the overloading is greater if both tones are strong; in both situations overloading appears to overlie and to obscure interference.

9. The form of the intensity function for the cochlear response is related both to interference and to overloading. The earlier process that gives transformation and interference is considered to cause the function to depart from a simple form and approach a limit. A later process, which is regarded as overloading proper, causes the function to pass through a maximum and bend downward. The inversion of the function at the extreme intensities is regarded as a momentary exhaustion of an essential process, perhaps the process of potential generation itself. The injury that follows overstimulation of the

ear may be an irreversible stage of this exhaustive action.

10. From the results on interference it is possible to divide the determinants of sensitivity as shown in the cochlear response into two types, and to infer the characteristics of each. They are the peripheral conditions, which precede the interference process, and some central condition, which intervenes between interference and the generation of the cochlear potentials.

The peripheral conditions include mechanical properties of outer, middle, and inner ears, and probably of the hair cells themselves. They determine the general form of the sensitivity function, except at the two ends of the frequency scale. The central condition possibly appears in the process of generation of electrical potentials. Its effect is that of a broad band-pass filter: it discriminates against the extremely low and high frequencies.

Three Mechanisms of Hearing by Electrical Stimulation

R. CLARK JONES,* *Research Laboratory of Physics,*
S. S. STEVENS, *Department of Psychology,*
AND
M. H. LURIE, *Department of Otology, Harvard Medical School,*
Harvard University, Cambridge, Massachusetts

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It was previously shown that when an alternating electric current is passed through the head via a salt solution in the external ear canal, a *normal* observer hears a tone which is related to the stimulus by a *square law*, that is to say, if the sound heard is interpreted in terms of motion of the tympanic membrane, then the displacement of the membrane is proportional to the square of the instantaneous voltage. The writers suggested the hypothesis that the cavity formed by the middle ear acts as an electrostatic transducer. This hypothesis accounted for the square-law response and for certain quantitative aspects of the results. This hypothesis had to meet the objection, however, that persons without tympanic membranes are able to hear by electrical stimulation. In order to determine the relation between the hearing of normal and operated ears, twenty ears lacking tympanic membranes were stimulated electrically. Eleven ears heard pure tones corresponding in pitch to the frequency of the applied voltage; seven heard a buzzing noise whose character was roughly independent of the stimulating frequency. Examination showed that the pure-tone response in the operated ears was purely *linear*, in contradistinction to the quadratic response of normal ears. Hence, under electrical stimulation normal and operated ears hear by means of two distinctly different mechanisms. The square-law response in normal ears is apparently mediated by an electrostatic action in the middle ear. The linear response in operated ears may or may not be the inverse of the cochlear microphonic. Evidence is presented that direct stimulation of the auditory nerve occurs in those ears which bear a buzzing noise. This result is consistent with a "place theory" of hearing, and it suggests that the "frequency theory" of hearing is untenable even for low frequencies.

INTRODUCTION

THE production of an auditory sensation by the passage of an electric current through the head constitutes the phenomenon of *electrophonic perception*. In the 140 years since this phenomenon was first explored by Volta,[1] the host of experimenters who have observed the auditory effects of electric currents have contrived a variety of theories to explain electrophonic perception. No lack of ingenuity is shown in these hypotheses and guesses: Almost every conceivable mechanism has been proposed. But no single explanation has yet proved adequate to all the observations. It now appears that the difficulty resides precisely in the fact that in the past only *single* explanations—unique causes—have been sought.

Taken in conjunction with our previous work, the evidence of the present experiment indicates that not one but three different mechanisms may be involved in electrophonic perception. Under different experimental conditions a sensation of hearing can be elicited by means of three quite different processes of electromechanical or electroneural transduction.

The need for hypothesizing three mechanisms is shown by the following phenomena. (1) When an alternating electric current is passed through a normal ear via a salt solution in the external ear canal, the subject hears according to a "square law."[2] He hears a tone an octave above the frequency of the stimulating current. He can, however, be made to hear a tone corresponding to the fundamental frequency of the alternating current by means of a polarizing direct current simultaneously applied. (2) Certain ears, from which the ear drum and ossicles have been removed in such a way that an electrode can be placed directly in contact with the external surface of the inner ear, hear according to a "linear law." The tone heard corresponds to the fundamental frequency of the alternating current and no second harmonic is detectable. The electrophonic effect in these ears is unaffected by a polarizing direct current. (3) Other ears whose

* National Scholar, Harvard University.

[1] A Volta, "On the electricity excited by mere contact of conducting substances of different kinds," Trans. Roy. Soc. Phil. **90**, 403–431 (1800).

[2] S. S. Stevens and R. Clark Jones, "The mechanism of hearing by electrical stimulation," J. Acous. Soc. Am. **10**, 261–269 (1939).

drums and ossicles have been removed hear only a noise when a sinusoidal current is passed through them. This noise remains essentially the same in character regardless of the frequency of the stimulating current.

In a previous paper[2] we demonstrated the first of these phenomena, namely, that the response of the normal ear to alternating current follows a square law. The subject hears the second harmonic of the stimulating frequency, unless a polarizing d.c. voltage is simultaneously applied. With this polarizing voltage the subject hears the fundamental frequency as well as the second harmonic. The perceived loudness of the second harmonic is independent of the magnitude of the polarizing voltage, but the loudness of the fundamental is roughly proportional to the impressed d.c. voltage.

Hence the normal ear responds to alternating current in precisely the same manner as does an electrostatic transducer, such as a condenser receiver. We proposed the hypothesis that the middle ear acts as such a receiver: the ear drum comprises one "plate" of the condenser and the opposite wall of the middle ear, the promontory, comprises the other "plate." This hypothesis accounts for the electrophonic effect as it is observed in normal ears and leads to certain quantitative predictions regarding the threshold of sensitivity which are borne out reasonably well by experiment.

The existence of a square-law response had been suggested[3] prior to our experiment, but the authors attributed the fact to some hypothetical electrostatic mechanism in the *inner ear*. Thus Hallpike and Hartridge[4] summarize the previous arguments with the statement that "the possibility that the tympanic membrane and ossicles are concerned in the production of the phenomenon has been excluded by Perwitzschky (1930) and by Gersuni and Volokhov (1934), who describe its persistence following the surgical removal of these structures." The assumption there is that one and the same mechanism is involved in hearing by normal and operated ears.

[3] A. Arapova and G. Gersuni, "On the frequency of alternating current and the pitch of the tone during electrical stimulation on the auditory apparatus," Tech. Phys. U. S. S. R. **5**, 447–462 (1938).

[4] C. S. Hallpike and H. Hartridge, "On the response of the human ear to audiofrequency electrical stimulation," Proc. Roy. Soc. London **B123**, 177–193 (1937).

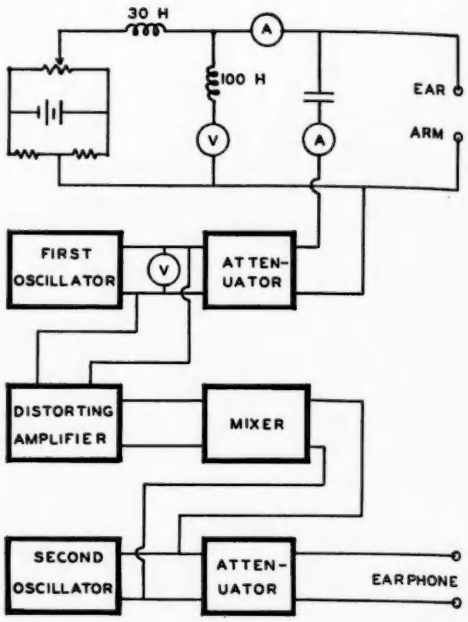

FIG. 1. Diagram of the circuit. The oscillators are General Radio type 613-B's; the amplifier is a General Radio type 514. The upper half of the diagram shows the circuit used for electrical stimulation of the ear, and the lower half shows the mixer and the apparatus used to apply a simultaneous acoustical stimulus.

However, it had not been demonstrated that hearing under these two conditions is the same "phenomenon." Among other things, the following experiments show that the two phenomena are not the same in the two kinds of ears—normal and operated—and that the middle ear, as we proposed earlier, is indicated as the locus of electrostatic transduction in the normal subject.

DESCRIPTION OF APPARATUS

The experimental work to be reported here was done in the Mosher Laboratory of the Massachusetts Eye and Ear Infirmary (Boston), and the subjects were patients in the Ear Clinic. The apparatus was similar to that used in the previous work,[2] but differed in detail because at the hospital the equipment had to be battery operated. The circuit diagram is shown in Fig. 1. The upper half of the diagram represents the apparatus which was used for the electrical stimulation of the ear. This circuit permitted us to stimulate the ear simultaneously with an a.c. and d.c. voltage, to vary these voltages independently and to measure independently the a.c.

and d.c. voltages and currents. The order of magnitude of the voltages and currents used was one volt and one milliampere, respectively.

The lower half of the diagram shows the circuit used for the simultaneous stimulation of the ear by an ordinary air-borne sound, and for the mixing of the outputs of the two oscillators so that their frequencies could be adjusted to near equality by the method of beats. It was desirable to obtain beats also between the second harmonic of the first oscillator and the fundamental of the second. Since the output of both oscillators was of low harmonic content, it was necessary to introduce some distortion into the output of the first oscillator. This was accomplished by the insertion of a distorting amplifier between the first oscillator and the mixer. The distorting amplifier consisted of a General Radio type 514 three-stage amplifier, with the last stage overbiased so that the tube was operating on the lower "knee" of its characteristic curve. Each of the oscillators was a General Radio type 613-B, with a frequency range 0–12,000 cycles.

The two signals were mixed acoustically, in order to eliminate coupling between the two circuits. The mixer consisted of two crystal earphones at opposite ends of a small chamber. A rubber tube led the sound from the chamber to the operator's ear, and prevented the acoustical beats from reaching the ear of the subject.

The ear electrode was a piece of No. 29 enameled copper wire, 10 cm long. At one end the wire was scraped and roughened, so that a tuft of absorbent cotton could be wrapped around the wire. When the electrode had been properly placed, it was clamped and held rigid by a device mounted on a block of wood curved to fit the head. The block of wood was strapped securely to the head. An electrode on the wrist provided the second connection to the body.

Experimental Procedure and Results: General

Twenty ears lacking tympanic membranes were examined in the course of the present work. Most of these ears had undergone radical mastoid operations with removal of the drum and ossicles of the middle ear. Nine ears heard pure tones corresponding in pitch to the frequency of the applied voltage; five heard a "buzzing" noise whose character was roughly independent of the stimulating frequency; two heard both the pure tone and the buzzing noise, although at different frequencies; and four heard nothing. Of these four ears, two had large amounts of scar-tissue filling the middle ear; one had no cochlea (removed by operation); and one had labrynthitis with complete loss of hearing.

The subject was seated with his back to the apparatus. The wrist electrode and head clamp were put in position, but the ear electrode was not inserted until we had demonstrated to the subject what we meant by a pure note, a high or low note, and a rising or falling note. We also demonstrated the phenomenon of beats by means of the acoustic mixer and trained the subject to follow the frequency of beats with his finger. The ear electrode with its tuft of cotton was dipped in physiologically normal salt solution, and was inserted through the external ear canal. It was usually placed on the oval or round window.

We then determined approximately the threshold of discomfort for a stimulating frequency of 1000 cycles. In our previous work[2, 5] on subjects with intact tympanic membranes, the maximum a.c. voltage which could be employed (usually between one and two volts) was that at which painful sensations were felt in the skin of the ear canal; these painful sensations were the same as are experienced in an ordinary electrical shock. In the present work, however, the limiting voltage was always that at which direct stimulation of some nerve became apparent. The nerves most frequently stimulated were the afferent fibers of the vestibular apparatus (the subject would become dizzy), and the efferent fibers of the facial muscles (the muscles on one side of the face would twitch). Occasionally the subject experienced a gustatory sensation, and as we shall see below, it appears that the auditory nerve itself was sometimes stimulated directly by the applied voltage.

If the threshold of discomfort was appreciably below one volt, we tried to find a placement of the ear electrode which would reduce these unwanted nerve stimulations. This search for a satisfactory electrode placement was coordinated

[5] S. S. Stevens, "On hearing by electrical stimulation," J. Acous. Soc. Am. **8**, 191–195 (1937).

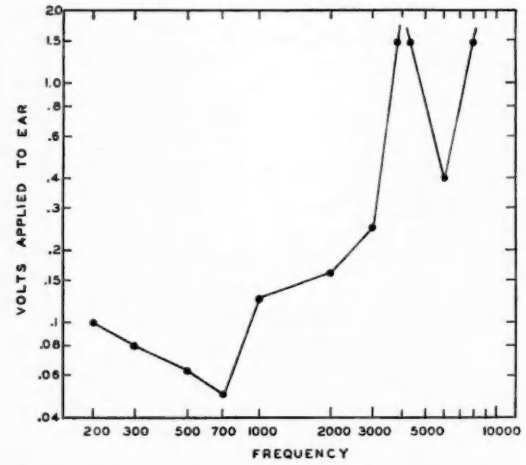

FIG. 2. Showing the electrical threshold of a subject who heard pure tones. The abscissa indicates the frequency of the applied voltage, and the ordinate the minimum voltage which was required to produce a perceptible auditory sensation. This particular curve is interesting because the voltages used are smaller than in any other case.

with a search for an auditory response. If no electrophonic perception was found at 1000 cycles, the frequency was shifted up and down the scale.

An auditory response was elicited in sixteen of the twenty ears examined. We shall present separately the results for the various types of response.

EXPERIMENTAL PROCEDURE AND RESULTS: PURE-TONE RESPONSE

The pure-tone response in operated ears had the following properties and identifying characteristics:

(1) When the frequency of the applied voltage was shifted slightly, the shift of the perceived pitch was always in the same direction. A small shift of frequency was readily detected.

(2) By means of the crystal earphone connected to the second oscillator an acoustical stimulus could be applied to the ear opposite the one being tested. When the electrical and auditory stimuli were alternately applied, the pitch of the electrical stimulus was called lower when and only when the frequency of the first oscillator was lower, and vice versa. A small difference of frequency was easily detected.

(3) The acoustical stimulus could also be applied to the ear being tested. In some cases the earphone was placed as close to the ear as the head clamp would permit, and in some cases the earphone was pressed to the cheek. The opposite ear was stopped off. Under these conditions, when the frequencies of the two oscillators differed by only a few cycles, strong beats were perceived and with his finger the subject could keep time with these beats. Whether or not the beat was being correctly followed could be checked by the experimenter listening to the beats from the mixer. No second harmonic was generated in the ear of the subject, for when the frequency of the second oscillator differed by a few cycles from the second harmonic of the first oscillator, as determined by means of the mixer, *no beats were heard.* Inasmuch as the initial purpose of this work was to settle this question of distortion in the operated ears, the search for such beats was made intensively, at many different frequencies. Nevertheless, no beats were obtained between the fundamental of the acoustical stimulus and any of the harmonics of the electrical stimulus with any of the eleven ears which heard pure tones under electrical stimulation.

(4) The only effect of applying a d.c. polarizing voltage to the operated ears was to lower the threshold of discomfort for a.c. stimulation. In particular, variation of the d.c. voltage had no effect on the situation with respect to beats.

The most important characteristics of the pure-tone response are the qualitative features presented above. The quantitative features are also interesting.

Some electrical threshold curves are shown in Figs. 2 and 3. The ordinate indicates the smallest applied voltage which gave a perceptible auditory sensation at the frequency indicated by the ordinate. Only five of the subjects were able to give consistently reproducible threshold curves; the others were not sufficiently good observers. The curves were reproducible only in the sense that they could be reproduced with a given electrode placement. A movement of the electrode would often give another curve which bore little resemblance to the first. An example of this is shown in Fig. 3 where the two curves were obtained with slightly different electrode placements.

No tonal response was obtained for stimulating frequencies below 100 cycles. There were two

difficulties. In the first place, the unwanted nerve stimulations usually became more prominent at low frequencies, and the threshold of discomfort was reduced. Secondly, the subjects described the same phenomenon that we had observed in our previous work on normal ears: Frequencies in the neighborhood of 100 cycles or below produce a feeling of deep vibration in the ear. Upon first experiencing this sensation, one is tempted to call it hearing. Closer examination, however, shows that the sensation is not hearing, but is probably of tactile origin. No pitch can be assigned to this vibration, and an acoustical stimulus of the same frequency in the opposite ear sounds quite different in character. The sensation is often described as uncanny.

Experimental Procedure and Results: Noise Response

The following characteristics identified the response obtained by those ears which heard a buzzing noise:

(1) When the frequency of the applied voltage was changed, the direction of shift of the perceived pitch bore no relation to the direction of the frequency change. Only rather large changes of frequency could be detected. Smaller changes in frequency were sometimes reported as changes in loudness. Changes in the stimulating voltage were sometimes reported as changes in pitch.

(2) When an acoustical stimulus of the same frequency was applied to the other ear, the subject remarked that the two tonal sensations were not at all alike. One subject had both eardrums removed. Under electrical stimulation he heard a pure tone in the right ear and a noise in the left. He described the two sensations as completely different.

(3) When an acoustical stimulus of nearly the same frequency was applied to the ear being tested, no clear beats could be obtained. One of our observers, however, was able to detect faint beats between the fundamentals of the two oscillators. These beats were evanescent, and could not be followed for more than a few beats at a time. In spite of their instability, however, there seemed little doubt that this observer heard the beats; for during the short periods when he was able to indicate their frequency, the beat of his finger agreed perfectly with the beats the operator heard in the mixer.

(4) The loudness of the noise increased very rapidly as the intensity of the stimulus was increased (see below). The response had the further property that the loudness usually decreased considerably as the stimulus was continued. This tendency for the sound to fade away made it difficult to make quanitative measurements of its loudness as a function of the stimulating voltage.

(5) The buzzing noise was usually heard only at stimulating frequencies between zero and 250 cycles. In one case, however, the noise was heard over the entire spectrum, 0–12,000 cycles.

As the stimulating frequency was reduced below about 50 cycles, the noise assumed a rhythmical quality, and at low frequencies of the order of ten or twenty cycles, it was described as being intermittent like the chirp of a cricket. At all frequencies higher than about 100 cycles, its character did not change appreciably as the frequency was changed. The one subject who perceived the noise over the entire range indicated that the sound did not change greatly in quality as the frequency was changed.

A rough estimate of the way the loudness of the noise varied with the applied voltage was obtained in the following manner. An acoustical stimulus was applied simultaneously to the opposite ear, and the subject was asked to adjust

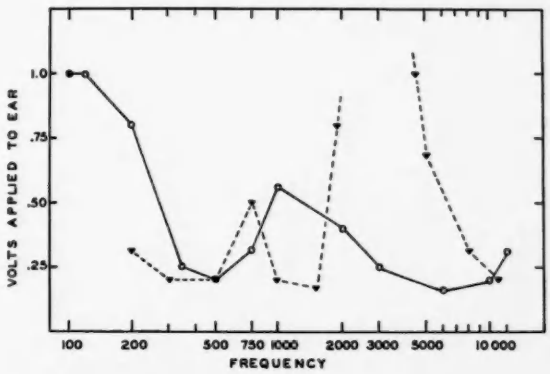

Fig. 3. Showing the electrical threshold of another subject who heard pure tones. The coordinates are the same as in Fig. 2. The two curves were taken with the same observer and during the same period of examination, the only difference being that the electrode placement in the middle ear was different for the two curves. Each of the curves was reproducible as long as the electrode was not moved.

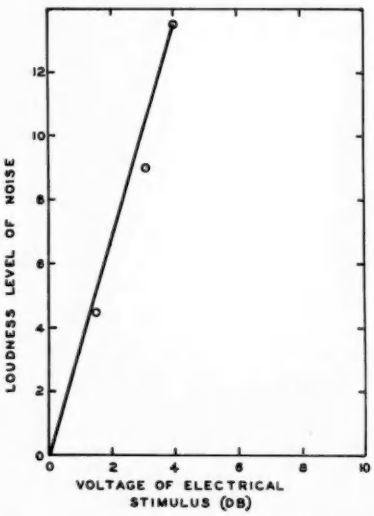

FIG. 4. Showing the rapid increase in the loudness of the noise as the stimulating voltage was increased. The data for this curve were taken as rapidly as possible in order to minimize the tendency of the noise to fade away as the stimulus was continued. Even so, noticeable fading did occur, and the curve would have been even steeper if the data could have been taken more rapidly.

the attenuator controlling its loudness until the loudness of the sound heard in each ear was about the same. This is, of course, a somewhat difficult estimate to make. Only three of the subjects were able to make this judgment consistently. With the electrical stimulus at 50 cycles, and the acoustical stimulus at 1000 cycles, the voltage of the electrical stimulus was set successively at 0.47, 0.56, 0.67 and 0.75 volt; the corresponding settings on the attenuator (acoustical stimulus) were 13.5, 9.0, 4.5 and 0.0 decibels (1.5-db steps). The total change in the voltage of the electrical stimulus corresponds to a change of 4 db. Hence a change of 4 db in the voltage of the electrical stimulus produced the same variation in loudness as a change of 13.5 db in the acoustical stimulus. Fig. 4 shows these results in graphic form. These results were obtained with the applied voltage increasing. The loudness of the noise decreased even more rapidly as the voltage was decreased. In the example just given, the noise vanished completely when the voltage was reduced from 0.75 to 0.40, although the noise could be heard clearly at 0.40 volt when the stimulus was removed for ten or fifteen seconds and then reapplied.

With another subject, and with both the electrical and acoustical stimulus at 100 cycles, the applied voltage was again set at 0.47, 0.56, 0.67 and 0.75 in that order, and the attenuator settings required to make the loudness equal in the two ears were again separated by intervals of 4.5 db.

A feeling of vibration at low stimulating frequencies was also noted by those who heard the buzzing noise; but this effect did not interfere with the measurements just described because the character of the noise was such that it was not confused with the feeling of vibration. The various extraneous nerve stimulations were about the same with subjects who heard tones and those who heard the buzzing noise. In most cases, the subjects would not tolerate a voltage much in excess of the voltage which would just produce a noticeable feeling of discomfort. One of our subjects, however, was quite willing to accept voltages much above the threshold of discomfort. The threshold of hearing (noise) and of discomfort are shown as a function of frequency in Fig. 5. This case is not typical, because the threshold of discomfort usually decreased at low frequencies. It will be noted in Fig. 5 that at 200

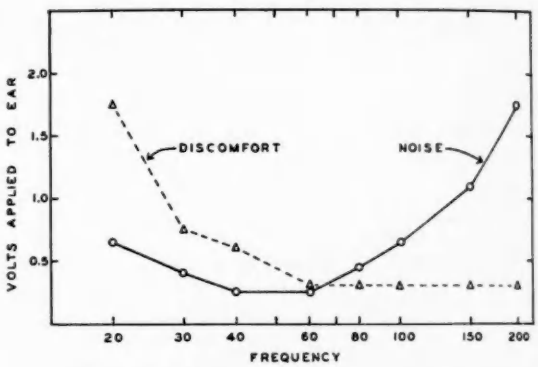

FIG. 5. Showing how the relative positions of the threshold for discomfort and the threshold for hearing the noise are dependent on the frequency of stimulation.

cycles the threshold of hearing represents a voltage nearly six times that of the threshold of dizziness; at this voltage the subject experienced a feeling of nausea as well as dizziness.

With no a.c. voltage, peculiar effects could be produced by changing the d.c. voltage. In one case, a noise was produced as the d.c. voltage on the ear became more positive, and the subject became dizzy when the potentiometer was turned in the other direction. This particular effect was

independent of whether the voltage itself was positive or negative; it depended only on the way the voltage was changing. In another case, bell-like sounds were heard with one direction of change, but nothing was heard with a change in the reverse direction.

Ears Hearing Both the Noise and the Pure Tone

Two of our subjects heard the buzzing noise at low frequencies, and pure tones at higher frequencies. The first of these presented the usual picture of hearing the buzzing noise at frequencies in the range 0–250 cycles, but he also heard pure tones in the frequency range 900–2100 cycles. The second case was more interesting, in that there was no gap between hearing the noise and hearing the tone. The transition occurred at 140 cycles, and was quite sharp: at 130 cycles, only the noise could be heard, while at 150 cycles, the noise had completely disappeared and the subject heard a pure tone with a pitch of 150 cycles. The threshold curve for this subject is shown in Fig. 6. It is interesting that there is no particular discontinuity at 140 cycles.

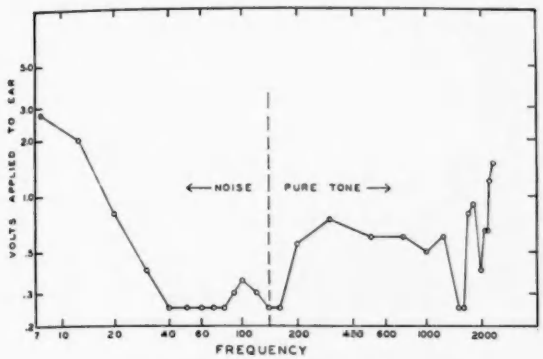

Fig. 6. Showing the electrical threshold of a subject who heard pure tones at stimulating frequencies above 140 cycles, and who heard a buzzing noise when the frequency of stimulation was below 140 cycles.

Interpretation of the Results

Taken in conjunction with the results of our previous work, the results here presented suggest that at least three different mechanisms are involved in the process of hearing by electrical stimulation. Under certain conditions, at least two of these mechanisms may be simultaneously involved.

The first mechanism

In our previous report it was demonstrated that in normal ears the response follows a square law; that is to say, if we interpret the sound heard in terms of movement of the tympanic membrane, then the displacement of the membrane is proportional to the square of the applied voltage. This square law was demonstrated by showing that when a simultaneous a.c. and d.c. voltage are applied between the ear and wrist electrodes, the perceived intensity of the second harmonic of the stimulating frequency is independent of the d.c. voltage, whereas the perceived intensity of the fundamental is proportional to the d.c. voltage. Furthermore, no harmonics higher than the second were found. This is sufficient to show that the major part of the response follows a square law. There were, however, definite indications of a small amount of linear response: (1) The loudness of the fundamental did not completely vanish at zero d.c. voltage. (2) The minimum in the loudness of the fundamental occurred at a d.c. voltage which made the ear about 0.08 volt positive, as measured from the voltage which made the d.c. current zero. A possible explanation of this residual linear component is pointed out below.

The square-law response can be explained on the basis of the electrostatic field set up within the middle ear by the applied voltage. The order of magnitude of the observed response is in reasonable agreement with the response to be expected from such an effect. This explanation thus assigns to the tympanic membrane the conversion of electrical energy into mechanical vibration.

The second mechanism

The results presented in this present paper show conclusively that the response of the ear when the tympanic membrane is absent follows not a square law, but a linear law, since no trace of a second or higher harmonic was found in the perceived sound under any conditions, and also since the perceived loudness of the fundamental was independent of the d.c. polarizing voltage.

This fact has important consequences. Since

the square-law response vanishes when the tympanic membrane is removed, it would appear that the square-law response is to be associated with the tympanic membrane. To be sure, this conclusion is not a logical necessity, although it does appear highly probable. At least it can be said that the ability of operated ears to hear by electrical stimulation does not compel us to abandon the hypothesis that the normal ear responds to an electric current by means of an electrostatic attraction of the eardrum.

As to the transducer involved when an ear lacking its tympanic membrane hears a pure tone under electrical stimulation we can offer no definite conclusion. The possibility of an electrostatic mechanism in the inner ear (suggested by Arapova and Gersuni[3]) seems excluded by the fact that the response follows a linear rather than a square law. On the other hand, the presence somewhere in the path of the a.c. of a membrane bearing a residual charge would provide for the linear conversion of electrical into mechanical energy. The existence of such a membrane has not been demonstrated, but Hallpike and Rawdon-Smith[6] have proposed that Reissner's membrane may be sufficiently polarized to account for the reverse effect, i.e., the conversion of sound into electricity.

This reverse effect leads to production of what is known as the cochlear microphonic. Over a limited range of intensities acoustic energy is transformed into electric energy according to a linear law.[7] This phenomenon occurs in man as well as in animals.[8] Now, the action of linear electromechanical transducers is often reversible and it is quite possible that the pure tones heard by operated ears are due to the inverse of the cochlear microphonic. This cochlear microphonic may be mediated by a polarized membrane, as suggested by Hallpike and Rawdon-Smith,[6] although it appears more probable that the electrical potential results from a compression of the hair cells of the organ of Corti. These cells are apparently analogous to piezoelectric crystals. Hence, we might venture the following hypothesis: The passage of an electric current through the inner ear sets into motion the hair cells of the organ of Corti. This motion tends to force the entire basilar membrane into vibration, but that portion of the membrane which, by virtue of the physical constants of the auditory mechanism, is preferentially responsive to the particular frequency of the stimulating current vibrates with the greatest amplitude. As a result of the mechanical vibration, nerve impulses are initiated in the appropriate fibers of the auditory nerve in the usual fashion. (Cf. Stevens and Davis,[7] Chapter 14.)

Regardless of the validity of this specific hypothesis, the fact remains that the response in operated ears follows a different law and is mediated by a different mechanism from the response in normal ears.

It should be pointed out that the existence of this linear response provides an explanation for certain effects encountered in our previous work.[2] There was evidence of a residual linear response which could not be abolished by variation of the polarizing voltage. It now appears that this small but persistent response to the fundamental frequency of the stimulating current may have been due to the same mechanism involved in hearing by operated ears. In other words, in certain normal ears there may be activated simultaneously both the linear mechanism (hair cells) and the nonlinear mechanism (tympanic membrane). This would account for the weak fundamental tone which we found to be out of phase with the fundamental produced when we polarized the tympanic membrane.

The third mechanism

In those ears that heard a noise whose character was substantially independent of the stimulating frequency it appears that the auditory nerve was directly stimulated by the electric current. Our reasons for subscribing to this hypothesis of direct stimulation of the auditory nerve are these:

(1) It is clearly possible to excite other nerves in the immediate vicinity of the auditory nerve. A variety of sensory and motor phenomena about the head and face of the subject testify to the

[6] C. S. Hallpike and A. F. Rawdon-Smith, "The 'Wever and Bray phenomenon.' A study of the electrical response in the cochlea with especial reference to its origin," J. Physiol. **81**, 395–408 (1934).

[7] S. S. Stevens and H. Davis, *Hearing* (Wiley, New York, 1938).

[8] A. M. Andreev, A. A. Arapova and G. V. Gersuni, "On the cochlear potentials in man," Bull. de Biol. et de Méd. Expér. **6**, 495–496 (1938).

ease with which different nerves can be excited under the conditions of our experiment. Most important, perhaps, is the evidence that the vestibular branch of the eighth nerve can be excited, i.e., the subject reports a sensation of dizziness. Since one part of the eighth nerve can be stimulated, it is reasonable to suppose that the other part is occasionally activated.

(2) A sensation of noise is what we should expect from electrical stimulation of the auditory nerve. This expectation is consistent with a "place theory" of hearing. The assumption is that the fibers stimulated by the electric current would be distributed more or less at random in the nerve, just as the activated fibers are distributed when a normal ear is subjected to a noise stimulus.

(3) The character of the noise changes only slightly or not at all when the frequency of the stimulating current is altered. This essential independence of frequency is consistent with a "place theory."

(4) The noise remains a noise at extremely low frequencies (less than one cycle per second). At 5 cycles, for example, the noise is intermittent, but the subject reports it to be the same noise heard at 100 cycles. It appears unreasonable to suppose that mechanical vibration in the cochlea at the rate of 5 per second would result in any such phenomenon as this.

(5) The loudness of the noise grows extremely rapidly as the voltage is increased. A slight increase in voltage is apparently sufficient to bring many new fibers into activity—many more additional fibers than would be activated by a corresponding increase in the intensity of an acoustic stimulus applied to a normal ear (see Fig. 4).

(6) The loudness of the noise resulting from a given voltage applied to the ear is rather unstable: The loudness usually declines rapidly during the first few seconds of stimulation. This kind of phenomenon is not uncommon when nerve fibers are stimulated directly by alternating current.

(7) The beats heard between the electrical stimulus producing a noise and an acoustic stimulus applied to the same ear are elusive and evanescent, when detectable at all. That such beats exist is not inconsistent with the hypothesis that the electric current stimulates the nerve directly, because we know from the phenomenon of binaural beats that alternate reinforcement and cancellation can take place between periodic excitations within the nervous system. (The binaural beats are also elusive and evanescent.) Beats between the electrical and the acoustic stimulus are evidence that the electrical stimulus is exciting the nerve fibers periodically and in synchronism with the electrical frequency.

(8) Finally, these observations confirm the results of Andreev, Gersuni and Volokhov[9] who observed a single case in which an operated ear heard a noise for all frequencies from 1.5 to 6000 cycles. The authors state that in this case the inner as well as the middle ear was damaged and that only stimulation of the auditory nerve occurred.

Relation to the Theory of Hearing

If our hypothesis of direct stimulation of the auditory nerve is correct, the facts of our present experiment have an important bearing upon the theory of hearing. It is generally admitted that a "place theory" in which perceived pitch is regarded as dependent upon *which* particular nerve fibers are stimulated best accounts for our perception of high tones. It has been argued, however, that the pitch of low tones may be ascribed to the ability of the brain to appreciate and differentiate the frequency of impulses carried by the auditory nerve. This is the essence of the "frequency theory" of hearing. According to this theory we should expect a pure tone to be heard whenever the fibers of the auditory nerve are stimulated periodically at, let us say, 50 times per second. But in the present experiment such stimulation produced not a pure tone but a noise of rather indeterminate pitch. It would appear, therefore, that the "frequency theory" may have to give way to the "place theory" even in the case of low tones.

The Problem of Thresholds

The question naturally arises as to what determines the type of response—noise, pure tone, or

[9] A. M. Andreev, G. V. Gersuni and A. A. Volokhov, "On the electrical excitability of the human ear. On the effect of alternating currents on the affected auditory apparatus," J. Physiol. U. S. S. R. **18**, 250–265 (1935).

both—obtained from an operated ear. We can give no definite answer to this question. There is apparently no very consistent relation between the type of response and the patient's audiogram as measured by air and by bone conduction. Of course, when the patient hears nothing by either air or bone conduction, he usually hears nothing by electrical stimulation, although there may be exceptions to this rule. Patients with "good" audiograms—those showing relatively slight loss due to the removal of the middle ear—can generally be made to hear a pure tone with a proper electrode placement, but some of these may hear nothing but a noise. It would require many more cases than the 20 explored by us to improve upon these rough generalizations.

We can, however, point to the principle factor making for complication in this problem. It is the matter of thresholds. Within a range of 10 or 20 db lie the thresholds for the pure-tone response, for stimulation of each of the two branches of the auditory nerve and for stimulation of various other nerves. Which of these will have the lowest threshold depends upon the position of the electrode, the "electrical geometry" of the tissues (paths of low and high resistance), the thresholds of the nerve fibers themselves and the frequency of the stimulating current. What the situation will be in a particular ear that has undergone a particular operation seems impossible to predict beforehand.

The relation of two of these thresholds to the frequency of the stimulating current is shown in Fig. 5. In general we should imagine at least four of these threshold curves rising, falling and crossing each other as a function of frequency. Then, whether a patient will hear a pure tone, hear a noise, feel dizzy, or suffer a facial twitch will depend, at each frequency, upon which threshold curve happens to be the lowest.

Results of the World's Fair Hearing Tests

J. C. STEINBERG, H. C. MONTGOMERY AND M. B. GARDNER
Bell Telephone Laboratories, New York, New York

(Received August 19, 1940)

The hearing test for musical tones which formed part of the Bell System exhibit at the New York and San Francisco Fairs is described. Average hearing acuity and the frequency of occurrence of various amounts of hearing loss are given as functions of age and sex. The relation of hearing acuity to place of residence, economic status and certain other factors is discussed briefly. Data from the two Fairs are used to determine the distribution of hearing acuity in the United States population, subject to certain stated limitations. Accuracy of the test is discussed in relation to ability of visitors to understand the test procedure, disturbing effect of background noise, and calibration of the test equipment. Certain results of the survey are expressed in terms of ear canal pressure and equivalent free field intensity, and on this basis a comparison is made with the results of other surveys of hearing.

WITH the opening of the New York and San Francisco World's Fairs in 1939, an opportunity became available for a survey of hearing in a large group of the United States population. One of the Bell System Exhibits consisted of a hearing test whereby visitors could test their hearing for tones of musical pitch. At the end of the test, each visitor was asked to permit an attendant to make a photographic copy of his hearing test card so that a study of the records might be made. Before making the copy, the attendant indicated by a check mark whether the visitor was male or female, colored or white, and to which of the five age groups, 10–19, 20–29, 30–39, 40–49, or 50–59, she judged him to belong. In all, some 550,000 photographic records were obtained, and it is estimated that about 80 percent of the visitors who tested their hearing for musical tones cooperated in the survey. A somewhat similar test for spoken words was also provided, but the survey was concerned principally with the results of the musical tones test.

The value and usefulness of such a large collection of records is dependent very directly upon the accuracy of the test. Therefore considerable attention was given to the calibration of the hearing test equipment and to the evaluation of factors which might affect the results of the test. There seems little doubt that the records accurately portray the hearing characteristics of that section of the population taking the tests.

One of the principal objectives of the study was to determine the hearing acuity and the prevalence of defective hearing in the United States population. The visitors who tested their hearing were not a representative sample of the population with respect to factors affecting hearing. Consequently a second objective was to determine the relation of hearing to such factors as age, sex, place of residence, economic status, etc. This information is necessary in order to apply the Fair data to the whole population or to specialized groups within the population.

It is believed that the two important factors,

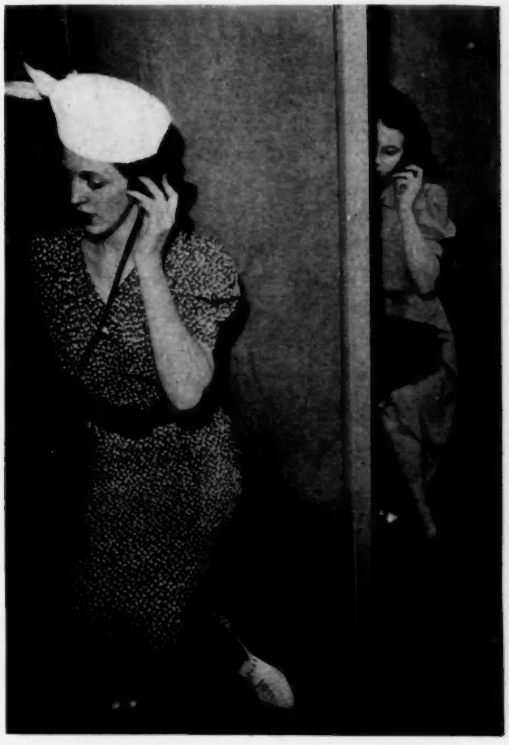

FIG. 1. Interior view of one of the hearing test booths.

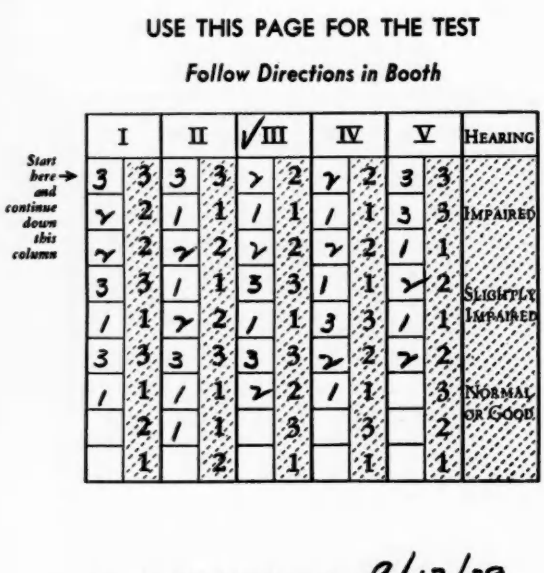

FIG. 2. Typical hearing test record as it would appear when illuminated from underneath.

age and sex, have been satisfactorily evaluated. Information on other factors although less complete, is sufficient to justify many applications of the data. In other applications, it is necessary to make reservations and these are described in the text.

DESCRIPTION OF THE TEST

The tests were made in sound-insulated rooms arranged to seat seven visitors, each partially screened from the others, as shown in Fig. 1. The test and suitable instructions were recorded on phonograph records and given through a telephone receiver which the visitor held to his ear. In the musical tone test a pure tone was sounded one, two, or three times, and the listener was instructed to write in a space on a form that was given him the number of times he heard the tone. For a given pitch, nine such sets of tones were sounded, each set fainter than the preceding one. When the tones became too faint to be heard, the listener could not write the number correctly, and thus a measure of his hearing acuity was obtained. This test was made with tones of five different frequencies in the following order: 440, 880, 1760, 3520, and 7040 cycles. A typical hearing test record is shown in Fig. 2.

The correct numbers, which appear in the spaces between the columns of Fig. 2, were printed on the back of the blanks in such a way that they would show through in these spaces when the blank was placed on a brightly illuminated glass shelf. The designations "normal or good," "slightly impaired" and "impaired," which show through opposite the eighth, fifth, and second steps, respectively, were intended to give a qualitative indication of hearing acuity. Thus the visitor could correct his own test and obtain an indication of his hearing acuity for each frequency.

A scale of hearing acuity for expressing the results of the tests was set up, using as a zero or reference level the mean test score of men and women at the Fairs in the age group 20 to 29. Hearing acuity is expressed as a hearing loss in the usual way, i.e., the departure in db of a given test result from the reference level. As shown in a later section, this reference level gives an ear canal pressure which corresponds closely to that for zero hearing loss on the 2A Audiometer.[1] Hence hearing losses given here are comparable in magnitude with audiometric measurements.

The range of the test is shown in Table I, which gives the hearing loss corresponding to each test step.[2] The range covered is 62 db in the first four columns and 48 db in the last.

TABLE I. *Hearing loss for each step in the musical tone test.*

COLUMN FREQUENCY	I 440	II 880	III 1760	IV 3520	V 7040
Step 1	52*	52	52	46	33
2	42*	42	42	36	27
3	32	32	32	26	21
4	22	22	22	16	15
5	14	14	14	8	9
6	8	8	8	2	3
7	2	2	2	−4	−3
8	−4	−4	−4	−10	−9
9	−10	−10	−10	−16	−15

* These tones were used in the instructions for the test.

[1] Some confusion has been occasioned by referring to the audiometer zero as normal hearing. Actually it is supposed to represent average normal hearing, where normal hearing refers to a range about the average value. A small hearing loss would not be considered abnormal.

[2] The hearing loss values given in Table I correspond to the actual tone levels in the test. Throughout this paper it is assumed that the threshold of an individual lies between the last level at which he correctly records the number of tones and the first level at which he does not, and in computing mean values the loss is reckoned mid-way between these two levels.

The voltage levels used at each frequency in the hearing test were selected so that the average young person would be able to hear the tones on the first six or seven steps, but would miss the last two or three.

Relation of Hearing to Age and Sex

The relation of hearing loss to age and sex may be summarized by giving the average hearing loss. More detailed information may be obtained from tables giving the frequencies of occurrence of different amounts of hearing loss. In this section, both types of data are given for men and women separately, in five age ranges.

Trends in average hearing

Average hearing as indicated by the mean hearing loss for men and women in five different age groups is shown in Table II. The number of tests used in obtaining the mean values is given in the right-hand column. For each age group, they were selected in a random manner from the New York and San Francisco tests, about two-thirds from New York and one-third from San Francisco.[3]

Certain trends in average hearing are evident in Table II. On the average, both men and women show increasing hearing impairment with increasing age. For high frequency tones, and especially at 3520 cycles, the effect is more pronounced in men than in women, but for low tones the opposite is true, although to a smaller degree.

Table II. *Mean hearing loss in db.*

	Age Group	Frequency 440	880	1760	3520	7040	Number of Tests
Men	10–19	1.0	.3	−.3	−1.2	−.4	4132
	20–29	0	−.2	−.1	2.0	1.5	3287
	30–39	1.4	1.3	2.3	8.2	7.7	3197
	40–49	3.7	4.5	7.0	17.7	16.8	4528
	50–59	6.8	7.7	12.1	25.6	24.0	1935
Women	10–19	.5	.2	−1.1	−4.4	−3.6	3417
	20–29	0	.2	.1	−2.0	−1.5	4208
	30–39	2.6	2.6	2.9	2.4	4.8	3978
	40–49	6.0	5.8	6.7	7.8	11.9	4369
	50–59	10.3	9.8	11.0	13.8	19.7	2538

[3] Several sampling procedures were used, all based on selection of tests by some arbitrary rule, such as taking four tests in order then skipping twelve. In general, the same rule was used throughout a whole day's tests. The days selected were well scattered throughout the season, and week days and week-end days were used in the proper proportion. A larger sampling proportion was used for the older age groups, to make the groups in the sample more nearly equal in size.

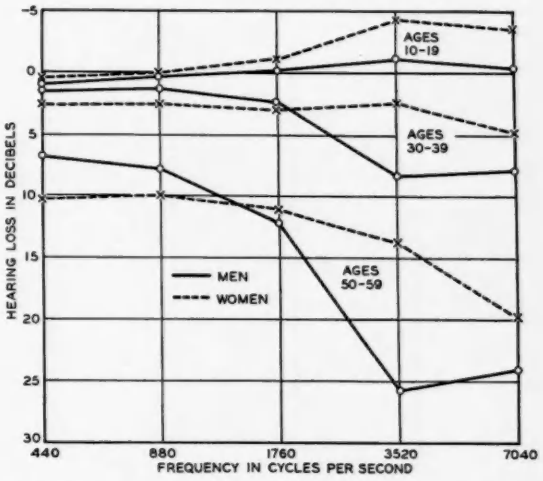

Fig. 3. Mean hearing loss in db for men and women in the youngest, middle and oldest age groups.

For the 1760-cycle tone there appears to be little difference between the hearing of men and women. These trends are shown in graphical form in Fig. 3, for the youngest, middle and oldest age groups.

At the lower frequencies, the hearing of the youngest group in Table II is slightly poorer than that of the next older group. It is believed that this is due principally to the greater difficulty of the younger children in understanding the test and writing their responses on the test blank.

Table III shows the standard deviations of the hearing losses for single tests. They range in magnitude from 7 to 20 db, and tend to increase with increasing age and tone frequency. An exception occurs for the 440-cycle tone where the values are mostly larger than for the 880-cycle tone. Since previous surveys have not shown a tendency for the standard deviation to increase below 880 cycles, it seems likely that the present increase occurred because the 440-cycle tone was the first one in the test, and initial unfamiliarity produced a greater scattering of the results. Carelessness in holding the receiver snugly against the ear would produce a similar scattering of results at this frequency.

The mean hearing losses and standard deviations for the older groups for the two high frequency tones would be somewhat larger were it not for the restricted scale of the test. At 3520 cycles the range from zero is only 46 db, and at 7040 cycles, only 33 db. In computing means

and standard deviations all results lying beyond the range of the test were grouped one test step beyond the extreme value included in the test.

Distribution of hearing loss

In addition to giving the trends in average hearing, the hearing test scores afford a means of determining the frequency of occurrence of different amounts of hearing loss. A convenient method of presenting the occurrence rates of different degrees of deafness is by means of curves or tables showing cumulative distributions. Table IV shows such distributions for the 35,589 test scores that were used in calculating the mean hearing loss values of Table II. It is arranged to show, for each of the five tones, the cumulative distributions separately for men and women in the five age ranges. It gives the percentage of tests failing to show a correct response at the test step indicated, or the percentage of individuals having a greater hearing loss than that corresponding to the indicated step. For example, the table shows that only 0.7 percent of the men in the 20–29 age group have hearing losses greater than 32 db for a 1760-cycle tone, while 11 percent of those in the 50–59 group have this much loss.

Zero hearing loss falls between steps 7 and 8 for the first three tones, and between steps 6 and 7 for the last two tones. The last step corresponds to very good hearing, and individuals able to hear this step have hearing acuities at least 10 db better than average. Some 10 or 15 percent of the youngest age group, but only 1 or 2 percent of the oldest group, were able to hear the tones on the last step. For the tone of lowest frequency, there were seven young persons for every older person who could hear the last step, but at 7040 cycles, there were 18 young persons for every such older person.

TABLE III. *Standard deviation of hearing loss in db.*

	Age Group	Frequency 440	880	1760	3520	7040
Men	10–19	10.9	8.7	9.4	12.7	14.2
	20–29	8.5	7.1	8.7	13.8	14.5
	30–39	10.1	7.9	9.9	16.6	16.2
	40–49	11.6	11.0	13.4	19.3	17.0
	50–59	12.1	12.6	15.8	19.2	15.1
Women	10–19	10.6	8.6	8.6	9.4	11.6
	20–29	9.3	7.9	8.5	10.1	11.8
	30–39	11.1	9.9	10.2	12.1	13.8
	40–49	12.4	11.8	11.9	14.1	15.8
	50–59	14.0	14.0	13.9	16.2	15.4

The tabular data for the age groups 20–29 and 50–59 are shown graphically in Fig. 4 for four of the tones, beginning at 880 cycles.[4] The curves are cumulative distributions and the ordinate gives the percentage of individuals having hearing losses greater than the value indicated by the abscissa.[5] At 880 cycles hearing losses in excess of a given amount tend to be more prevalent among women than among men. At 1760 cycles the distribution curves for men and women are much the same. At the two higher frequencies, the prevalence of deafness in excess of a given amount is greater among men than among women.

A hearing loss of 25 db at frequencies up to 1760 cycles begins to be a handicap. The individual will usually be aware of such an impairment, and will experience difficulty in understanding speech under conditions of public address, such as in the church or theater or around the conference or dinner table. The distribution curves show that only about 1.5 percent of the young people taking the test, or three out of 200, have a hearing loss of 25 db or more for tones of these frequencies. In the oldest age range almost ten times as many, or every seventh person, shows this much impairment.

A hearing loss of 45 db for frequencies up to 1760 cycles will usually make it difficult to hear direct conversation even when the speaker is about two or three feet away. Individuals with this much loss usually need some sort of hearing aid. Table V gives the percentages of various groups that have losses in excess of 25 db and 45 db at the various frequencies.

Acuity for the two high frequency tones is less important than for the low tones for understanding speech, so that a loss for the high tones is not such a serious handicap. High tone deafness is of particular interest, however, to the extent that it is indicative of a progressive condition which may later involve tones of lower frequency. It is striking to note that, of the people taking

[4] The distributions for 880 cycles may be used for 440 cycles as well, it being assumed that the small difference shown in Table IV is due to practice.

[5] The ordinates are shown on an arithmetic probability scale, which has the property that a normal distribution plots as a straight line whose slope is proportional to the standard deviation of the distribution. It is convenient because it shows the small values more accurately and because on this scale the standard errors of the ordinates are approximately equal in all parts of the range.

TABLE IV. *Distribution of hearing loss. Percentage of tests failing to show a correct response at the test step indicated, based on 23,320 tests from the New York Fair and 12,269 tests from the San Francisco Fair.*

FREQ.	TEST STEP	HEARING LOSS	MEN AGE GROUP 10–19	20–29	30–39	40–49	50–59	WOMEN AGE GROUP 10–19	20–29	30–39	40–49	50–59
440	1	52*	—	—	—	—	—	—	—	—	—	—
	2	42*	—	—	—	—	—	—	—	—	—	—
	3	32	2.7	1.2	2.5	4.1	5.4	2.5	1.5	3.5	5.4	9.9
	4	22	4.6	2.1	4.2	7.1	9.8	4.1	3.0	6.1	10.7	17.9
	5	14	9.2	5.3	8.3	13.9	21.2	8.4	6.6	11.8	20.9	33.5
	6	8	16.0	11.0	14.7	22.3	32.1	15.2	12.1	19.4	30.9	44.3
	7	2	40.0	32.3	38.7	46.7	60.9	37.7	33.5	43.3	56.2	68.9
	8	− 4	67.1	67.9	71.3	77.6	85.7	64.7	65.1	72.9	81.8	88.1
	9	−10	86.2	92.2	93.6	95.1	97.8	84.9	90.7	94.0	96.2	98.0
880	1	52	.4	.1	.1	.7	1.4	.4	.2	.8	1.2	1.9
	2	42	.7	.2	.4	1.8	3.2	.6	.5	1.4	2.6	5.3
	3	32	.9	.6	.9	3.2	5.4	1.0	1.1	2.1	4.1	7.6
	4	22	2.1	1.3	2.3	6.8	12.0	2.2	2.1	4.4	8.7	16.1
	5	14	4.9	2.9	5.5	13.5	21.6	4.6	4.3	8.5	16.0	27.1
	6	8	10.3	7.4	11.7	23.0	33.6	9.3	8.2	15.3	26.4	40.9
	7	2	34.3	30.2	37.5	49.3	62.3	32.5	29.9	40.8	55.1	68.0
	8	− 4	77.5	76.1	81.8	87.3	92.3	77.4	78.1	84.0	89.6	93.3
	9	−10	88.9	93.8	96.2	97.6	98.9	90.6	95.0	97.4	98.2	98.8
1760	1	52	.5	.2	.4	1.7	4.3	.2	.2	.6	.8	1.6
	2	42	.6	.4	.7	2.9	6.3	.5	.4	.9	1.8	3.9
	3	32	1.0	.7	1.5	5.6	11.0	.8	.8	2.1	3.8	8.3
	4	22	1.9	1.4	3.5	11.0	19.5	1.4	1.9	4.4	8.8	16.6
	5	14	5.0	5.7	9.5	20.3	33.4	3.8	4.9	9.4	19.5	31.9
	6	8	11.3	11.5	18.3	32.9	47.8	7.9	10.7	17.8	32.9	47.5
	7	2	31.5	31.8	40.3	55.6	69.1	26.7	30.7	43.0	58.7	70.4
	8	− 4	63.5	63.6	74.6	82.8	90.2	60.1	66.4	77.2	85.1	91.2
	9	−10	85.1	89.4	93.3	95.6	97.9	85.9	91.0	94.5	97.3	97.8
3520	1	46	1.8	2.5	5.8	15.3	25.6	.2	.6	1.5	3.0	6.7
	2	36	2.3	3.9	8.8	20.4	33.8	.4	1.1	2.3	5.0	10.7
	3	26	3.8	6.3	13.6	29.9	45.8	1.0	1.9	4.7	9.6	19.8
	4	16	7.8	11.7	23.5	44.8	61.7	2.9	4.5	10.4	21.9	36.2
	5	8	13.3	20.2	36.1	60.1	75.5	6.6	10.2	21.2	37.7	54.7
	6	2	26.7	35.7	54.0	73.9	86.8	16.4	21.9	38.0	56.2	72.3
	7	− 4	54.5	64.1	76.9	88.8	95.3	45.0	52.5	68.6	81.7	89.7
	8	−10	78.4	85.8	93.2	97.0	99.2	71.8	81.2	91.3	95.9	98.0
	9	−16	90.6	95.6	98.1	98.9	99.7	89.0	95.5	98.2	98.9	99.3
7040	1	33	6.3	7.8	16.1	35.0	53.1	1.6	2.7	8.4	20.1	38.3
	2	27	7.1	8.9	18.1	37.5	56.6	2.0	3.3	9.6	22.7	41.1
	3	21	8.7	10.7	20.8	41.8	60.8	3.0	4.4	11.9	27.2	45.9
	4	15	13.2	15.3	28.6	50.9	70.5	6.2	7.9	19.1	38.0	57.7
	5	9	20.6	23.8	38.6	61.7	78.4	13.8	15.7	31.1	50.7	70.2
	6	3	32.5	35.7	52.4	72.6	86.4	24.1	28.2	47.2	64.9	82.1
	7	− 3	49.4	54.7	70.4	85.0	94.0	41.3	49.3	67.7	81.3	92.6
	8	− 9	70.2	75.6	85.0	93.8	97.5	64.0	72.3	86.0	92.7	97.6
	9	−15	82.7	88.1	94.5	97.6	98.9	80.0	87.9	95.1	97.5	99.1
	No. of Tests		4132	3287	3197	4528	1935	3417	4208	3978	4369	2538

* These tones were used for the instructions for the test.

the hearing test at the World's Fairs, some 6 percent in the 20–29 year age range showed a hearing loss in excess of 25 db for the 7040-cycle tone. In the oldest age range half of the people showed such a loss. It is likely that an even larger proportion would be found in a random sample of the population, for, as will be discussed in a subsequent section, it is believed that the people taking the test at the Fair represent an economic status that is average or better, and there are indications of a greater prevalence of hearing defects in the lower economic groups.

The estimation of age

To determine the accuracy of the age estimates 267 test blanks of members of Bell Telephone

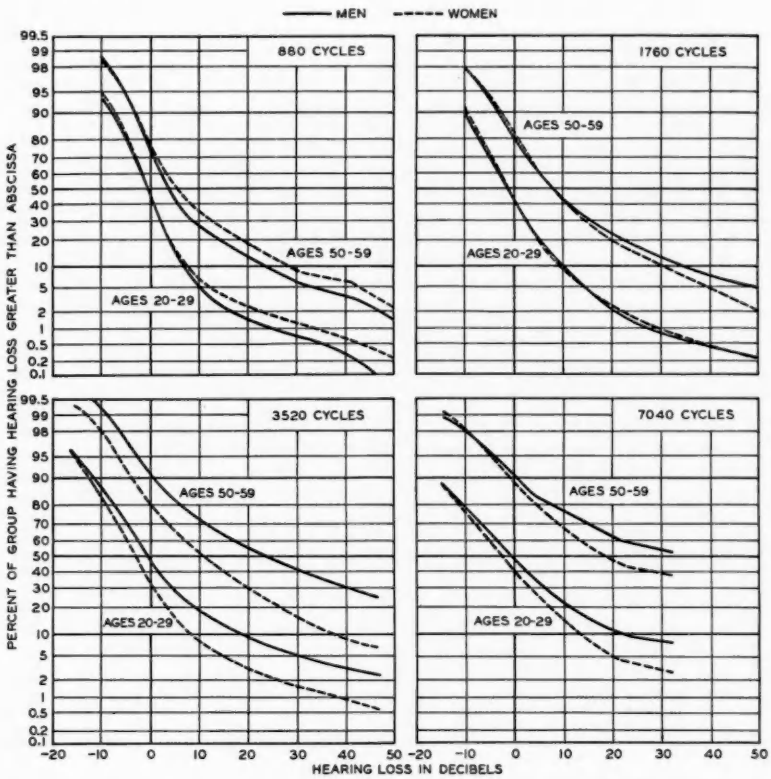

FIG. 4. Percentage of people in a given age and sex group having a hearing loss greater than any given value.

Laboratories families were examined and the real age compared with the estimated age indicated on the blank. In most cases the attendant who made the estimate was not aware that the individual was a member of the Bell System. It was found that 62 percent of the real ages were within the estimated 10 year age group, while 83 percent were not more than 3 years outside of the group and 96 percent were not more than 8 years outside of the group. There was a general tendency to estimate high for young people and low for older people, but it appeared that the average real age in any estimated age group was within one or two years of the midpoint of the group. The oldest group at the New York Fair included ages 50–59 with an average of about 54, while the oldest group at San Francisco included all over 50 with an average of about 60.

RELATION OF HEARING TO OTHER FACTORS

There is no doubt that hearing is related to factors other than age and sex. This was shown by plotting test results on a control chart;[6] definite evidence of lack of statistical control was found even after making corrections for age and sex differences. This section contains a very brief summary of the results of a study of these relationships.[7]

Comparison of tests obtained at the New York and San Francisco Fairs revealed no appreciable differences at the three lower frequencies. At the two higher frequencies there was little difference for women, but men tested at New York had some 3 db greater acuity than men at San Francisco. There appeared to be little difference at either Fair between the hearing of those living in the city where the Fair was held and visitors from outside.

There was evidence that people of high economic status had better hearing than those of

[6] W. A. Shewhart, *Economic Control of Quality of Manufactured Product* (D. Van Nostrand, 1931).

[7] The material of this section will be treated in more detail in a paper by the same authors in Bell Sys. Tech. J., October, 1940, issue.

low economic status. This accords with other studies that have been reported. No appreciable difference between right and left ears was found. No trends of hearing acuity with time of day could be discovered in any age group.

Distribution of Hearing Acuity in the United States Population

The tests at the two Fairs constitute a large cross section of the United States population. It is not a representative cross section in certain respects, the most important of which are described below. It is believed that by taking into consideration the limitations mentioned below an estimate of the distribution of hearing acuity in the United States population can be obtained which is sufficiently accurate for most practical purposes.

The two Fairs taken together probably represent a good geographical cross section of the country, except that areas near the Fairs were too heavily represented. However, since no pronounced differences in hearing were found for those living near the Fairs, the geographical sampling may be regarded as fairly satisfactory. Although it is quite possible that sections might be found in which hearing differed markedly from the Fair values, it is unlikely that such areas would be extensive enough to affect the over-all result.

With regard to economic status, intelligence, and amount of education, the Fair groups were judged to be somewhat above average, and probably representative of the upper two-thirds or three-quarters of the population If hearing is related to such factors, as seems probable, the hearing of the population is not quite so good as indicated by the Fair tests.

The portion of the distribution curves relating to large hearing losses at the three lower frequencies must be accepted with reservations on account of the possible biasing effect that would arise if people who were aware of a substantial hearing impairment were more likely to take the test than normal hearing people. A bias in the opposite direction would arise if people with impaired hearing were less willing to allow records to be made of their tests. It is not likely that mean values would be affected to any important extent, nor the portion of the distribution curves below, say, 35 db hearing loss. Nor is it probable that the distribution curves for the two highest frequencies would be much affected, for people are less likely to be aware of losses at these frequencies.

The distribution of ages at the Fairs was quite different from that in the population. The first step in allowing for that difference was made by recombining the distributions of hearing loss for various age and sex groups shown in Table IV, weighting each according to the size of the group in the population. The resulting distributions are shown in Fig. 5, and apply to the age range 10–59 years.

In a similar manner, the figures in Table V for the incidence of hearing loss of 25 db or more were weighted according to the size of the groups and combined, leading to the values given in the first line of Table VI, for the age range 10–59. This process can be extended to include the whole age range as follows. It is assumed that the incidence for ages under 10 is the same as in the 10–19 group. This may not be strictly true, but is a sufficiently good approximation for this purpose. For ages above 60 a minimum estimate was obtained by assuming that the incidence of hearing loss is the same as in the 50–59 group, and a maximum estimate by assuming 100 percent incidence above age 60. The actual value may be expected to be somewhere between these limits, which are shown in Table VI.

Except for the reservations stated in the first four paragraphs of this section, it is believed that

Table V. *Percentage of tests with hearing loss greater than 25 and 45 db.*

Age Group	25-db Loss Frequency[4] 880	1760	3520	7040	45-db Loss Frequency[4] 880	1760	3520
10–19 Men	1.7	1.6	4.5	8.0	.6	.6	1.8
Women	1.8	1.2	1.2	2.5	.6	.4	.3
20–29 Men	1.1	1.2	7.0	9.5	.1	.3	2.7
Women	1.8	1.6	2.2	3.5	.4	.3	.7
30–39 Men	1.8	3.5	15	19	.3	.6	6.0
Women	3.5	3.5	5.5	10	1.2	.8	1.6
40–49 Men	5.5	9.5	32	39	1.4	2.6	16
Women	7.0	7.0	11	24	2.1	1.5	3.0
50–59 Men	9.5	17	48	58	2.6	6.0	27
Women	13	14	22	43	4.0	3.0	7.0

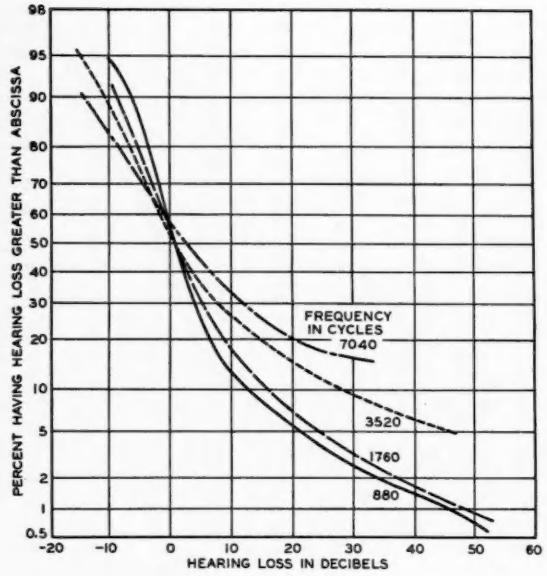

FIG. 5. Percentage of people, both men and women, in the age range 10–59 having a hearing loss greater than any given value.

the data of Figs. 3, 4, and 5 and Tables II, III, IV, V, and VI should apply fairly closely to the whole United States population. They may be applied to groups in the population who are not specialized in regard to any factor related to hearing. It would be unsafe to apply them to a group of very low or unusually high economic status, college graduates, unskilled laborers, foreign groups, etc. without further knowledge of the relation between hearing and the factor in which the group was unusual.

ACCURACY OF THE HEARING TEST[7]

Observation of people taking the tests at the Fair indicated that the great majority took the tests seriously and made a conscientious attempt to test their hearing. It was noticed that a very small percentage of the people, mostly in the youngest age group, altered their scores by filling in all of the missing numbers before having them photographed, thus giving a false appearance of a perfect test, but it is believed that the number of such scores was too small to affect the hearing loss distributions appreciably.

Some people undoubtedly secured poor scores in the tests on account of failure to understand the test, interruptions, or other causes not connected with hearing. A study of this factor was made by interviewing individuals who had almost completely failed the test. It was found that about two-thirds of this group failed the test because they were definitely hard of hearing. Also, as it happened, about two-thirds of this group submitted their test scores to be photographed; so that the number of recorded failures tended to be the same as the number that were actually hard of hearing.

The noise conditions under which the tests were given were quite favorable. The exhibit building was quiet due to the generous use of sound absorbent walls and carpeted floors. Parts of the building containing air-conditioning and other machinery were constructed on a separate foundation from the part containing the hearing test booths. Noise in the booths from external sources was nearly inaudible and it is probable that most of the disturbing noise was caused by people participating in the test.

To evaluate the disturbing effects of noise, about 150 members of Bell Telephone Laboratories who had taken the test at the Fair under routine conditions at various times during the season were retested at the Laboratories after the Fair closed. The same equipment and procedure were used, except that only one person was tested at a time under conditions free from any disturbing noise. The average improvement was 2.9 db at 440 cycles, 1.4 db at 880 cycles, 1.1 db at 1760 cycles and negligible at the higher frequencies. Since the test at the Laboratories was given last in every case, these shifts may have been partly due to improvement with practice and it can be seen that the disturbing effect of noise was quite small. This conclusion is in harmony with the results of sound level meter readings made in a booth while tests were in progress.

A complete calibration of the test equipment at each level used in the test was made twice during the season. In addition to this, careful

TABLE VI. *Percentage of people having hearing loss of 25 db or more.*

		Frequency[4] 880	1760	3520	7040
Ages 10–59		3.8	4.5	12	18
All ages	Minimum	4	5	12	18
	Maximum	11	12	18	22

TABLE VII. *Calibration of hearing test equipment at the reference level.*

	Frequency				
	440	880	1760	3520	7040
Reference voltage level across receivers—db above one volt	−104	−112	−115	−114	−76
705A receiver calibration—db above 0.0002 dyne per sq. cm per volt	133	134	133	134	98
Reference ear canal pressure level—db above 0.0002 dyne per sq. cm	29	22	18	20	22

check of the performance of various parts of the apparatus was made at frequent intervals. The engineers in charge of the equipment made a listening test at the beginning of each day. It is believed that the ten machines used at the two Fairs were alike in output, including the effect of receiver differences to within 2 or 3 db at all levels

HEARING TEST RESULTS IN TERMS OF PRESSURE AND INTENSITY LEVEL

In order to compare the results of the Fair tests with other data on hearing, calibrations have been made of the receivers that were used in the tests. This was done by measuring the pressure levels developed by the receiver at the opening of the ear canal for a small group of people, using a special search tube transmitter so designed that the tube could be inserted under the receiver cap into the opening of the ear canal. Such a calibration gives an ear canal pressure level in terms of receiver voltage levels. The authors are indebted to Mr. W. A. Munson of these Laboratories for the calibrations. They are preliminary in character and may need modification in the light of subsequent studies.

With the aid of the receiver calibrations just described, ear canal pressure levels may be calculated from the receiver voltage levels measured in the tests. Such calculations for the reference level or condition of zero hearing loss as used in this paper are shown in Table VII. The resulting reference ear canal pressure levels are plotted in Fig. 6. For comparison, the ear canal pressure levels corresponding to the zero hearing loss settings of the 2A and 6A audiometers[8] and the minimum audible pressure curve derived by Sivian and White[9] are shown. The audiometer curves and the Fair curve are based on ear canal pressures measured at the ear opening in the manner just described. The minimum audible pressure curve is based on pressures measured about 1 cm from the eardrum, which correspond more nearly to eardum pressure levels. The two types of measurements are undoubtedly quite comparable below 1000 cycles. For frequencies above 5000 cycles and possibly around 2000 cycles it is believed that the pressure at the ear opening is somewhat smaller than the corresponding eardrum pressure.

A comparison of the Fair data with data from two other surveys of hearing is shown in Fig. 7. One curve shows the mean threshold pressure for men in the 20–29 age group from the Fair data. Another gives values for the same age and sex group in a survey conducted in 1936 by the United States Public Health Service using the 2A Audiometer.[10] This curve is for a somewhat selected group, including only individuals who stated when the test was made that they believed their hearing was normal. The third curve is for members of Bell Telephone Laboratories in the same age and sex group, who were tested in 1931 with a 2A Audiometer.[11] In comparing these results, it should be remembered that differences may be due to three general causes. The groups of people tested may have differed in hearing acuity. The calibrations by which the ear canal pressures were established are subject to error, especially at high frequencies. The conditions of the test, including technique, concentration of subjects, receiver fit, and background noise,

[8] J. C. Steinberg and M. B. Gardner, "Auditory significance of hearing loss," J. Acous. Soc. Am. **11**, 270 (1940). In using the Audiometer it is customary to record as the hearing loss the lowest dial setting at which the tone is heard. Threshold would, on the average, be half a dial step lower than the recorded setting. Hence the curves given here for zero hearing loss are 2.5 db lower than those given for zero dial setting in the reference. See also reference 2.

[9] L. J. Sivian and S. D. White, "Minimum audible sound fields," J. Acous. Soc. Am. **4**, 288–321 (1933).

[10] W. C. Beasley, *National Health Survey, Hearing Study Series*, Bulletin 5, Table 3 (The United States Public Health Service, Washington, D. C., 1936).

[11] H. C. Montgomery, "Do our ears grow old?" Bell Lab. Record **10**, 311 (1932). Note that median values were given in this reference, differing slightly from the mean values used here.

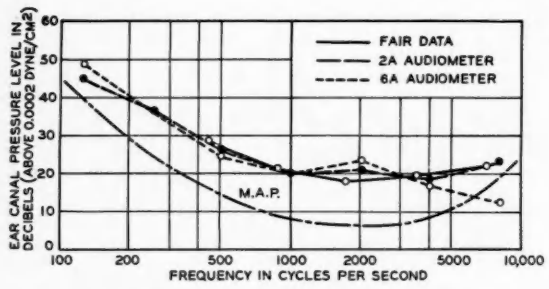

FIG. 6. Ear canal pressure level for certain reference conditions.

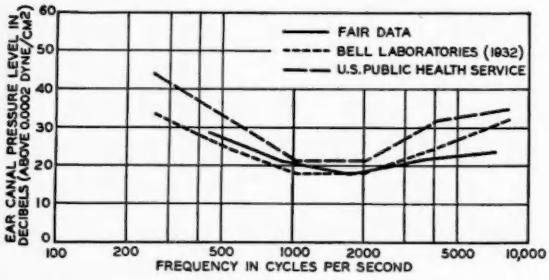
FAIR DATA
BELL LABORATORIES (1932)
U.S. PUBLIC HEALTH SERVICE
EAR CANAL PRESSURE LEVEL IN DECIBELS (ABOVE 0.0002 DYNE/CM²)
FREQUENCY IN CYCLES PER SECOND

FIG. 7. Comparison of several surveys of hearing, giving mean ear canal pressure level for men aged 20–29.

were not alike in all cases. Considering all the possible causes of variation the curves seem to be in fairly good agreement.

In order to give a preliminary picture of the prevalence of deafness in terms of free field intensity and frequency, the distribution curves of Fig. 5 were converted into the contour lines shown in Fig. 8. For the low frequencies, this conversion was made by applying the differences between the minimum audible pressure and minimum audible field curves of Sivian and White[9] to the ear canal pressure levels of Table VII. For the high frequencies, the conversion was based on a free field calibration of the receivers. The free field intensity levels apply for the condition in which the observer faces the source and listens with both ears. The resulting contours purport to show the percentage of people in the population within the age range 10 to 59 years who cannot hear tones below the given level. The boundary lines forming the auditory sensation area represent a picture of the limits of useful hearing based upon earlier studies.[12] The solid portions of the contour lines represent the distributions obtained from the results of the Fair tests. The dotted portions represent extrapolations of the distributions beyond the intensity and frequency ranges used in the tests, and are of course speculative in character.

[12] H. Fletcher, "Auditory patterns," Rev. Mod. Phys. **12**, 47–65 (1940).

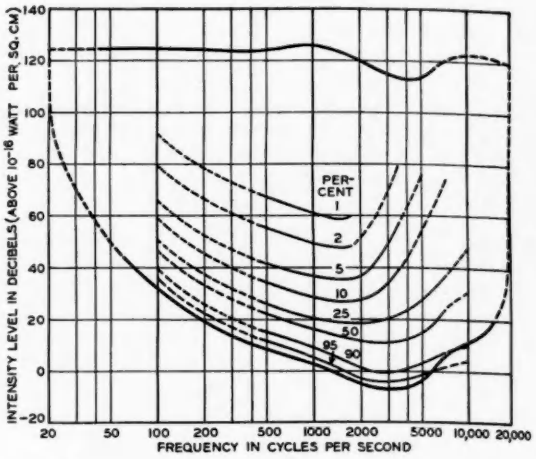

FIG. 8. Contour lines above which lies a given percentage of thresholds for the age group 10–59.

The extrapolations to lower frequencies are based on the shape of the contours below 1760 cycles and the high correlation that has been found to exist in individual audiograms for frequencies from 64 to 512 cycles.[13,14] The extrapolations to large hearing losses for the 3520- and 7040-cycle tones were made by extending the curves of Fig. 5 as suggested by comparison with the results of other surveys.

The contours show several interesting things. The range over which hearing acuity varies is quite uniform up to about 2000 cycles, and 90 percent of the group lie within a range of 30 db. Above 2000 cycles the range increases rapidly. Since most of the sounds met with in daily life have intensity levels greater than the 25 percent contour, fully three-fourths of the people can hear ordinary sounds throughout the frequency range from 100 to 10,000 cycles.

ACKNOWLEDGMENT

This survey was made possible by the cooperation of a large number of people in many parts of the Bell System. The planning, design,

[13] E. G. Witting and W. Hughson, "Inherent accuracy of repeated clinical audiograms," Laryngoscope **50**, 259 (1940).

[14] W. C. Beasley, "Correlation between hearing loss measurements," J. Acous. Soc. Am. **12**, 104–113 (1940).

and construction of the exhibit were shared by the American Telephone and Telegraph Company, the Western Electric Company, Electrical Research Products, Inc., and Bell Telephone Laboratories, Inc., and to them the authors are indebted. We wish to express our gratitude to the Pacific Telephone and Telegraph Company and the New York Telephone Company, for their efficient operation of the exhibits and for the large share which they had in obtaining the data used in the survey; to the tabulating and mathematical groups at the Laboratories for many hours of painstaking labor in treating the data; and to many of our associates whose suggestions and criticisms were a valuable aid in the analysis of the information. We also wish to express our appreciation to the large group of interested visitors to the Fairs whose participation in the hearing tests constituted the basic material of this survey.

Acoustical Society News

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New York, New York,
American District Telegraph Co., 155 Sixth Avenue.

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Pipestone, Minnesota,
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Eno Hall,
Princeton, New Jersey,
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University of California at Los Angeles,
Los Angeles, California.
Student.

Future Meetings

Fall 1940. Chicago. November 15, 16.
Spring 1941. Rochester, N. Y.
Fall 1941. New York. Oct. 24, 25. Joint meeting with the founder societies of The American Institute of Physics.
Spring 1942. Washington.

Notice of Amendment to By-Laws

AMENDMENT III. *Retiring Members.*—At the age of 65 years or more, any member or Fellow in good standing who has been a member for 5 years or more may at his own request be placed on the retired list and be exempt from further dues payments. Retired members shall have full membership rights except the receipt of Journals.

Acoustical Society of Japan

The Editor has just received the following letter concerning the activities of the Acoustical Society of Japan:

Dear Sir:

I wish to have the honor to present your Society the Journals of the Acoustical Society of Japan which have been trusted to me. Our Society was organized in April, 1936, and is continuing its activity, holding now about 500 members of various specialists who have interest in acoustics. They have earnest desire to have intimate connection with your Society in many points.

Yours respectfully,

Minoru Takada

Professor of Keijo Imperial University
Member of the Acoustical Society of Japan
Hotel Yamato, 3 W. 73 St., New York, New York

Enclosed were three copies of the *Journal of the Acoustical Society of Japan*, averaging about 50 pages each. Unfortunately for American readers, these are published in Japanese, so that only the figures and pictures look familiar. This Journal contains tables of contents of the various issues of the *Journal of the Acoustical Society of America* and the *Akustische Zeitschrift*. The address of the Acoustical Society of Japan is 31. Kamifujimae Hongoku (Rikagaku Kenkyujo), Tokyo City, Japan, and the President is Dr. Mishio Ishimoto, of the Tokyo Imperial University.

Current Publications on Acoustics

F. A. FIRESTONE

147 East Physics Building, University of Michigan, Ann Arbor, Michigan

Reviews of Contemporary Papers

MEMBERS who have occasion to translate important papers from foreign journals will be doing a real service to those members who do not read the foreign languages easily, if they will prepare a review of such articles for publication in this section.

These reviews should be of the nature of a lengthy abstract (500 to 1500 words) rather than a critique or appraisal, and should attempt to set forth in this limited space as much of the original author's contribution as possible. One or two figures may be included if desired.

Particular articles in French or German periodicals will be translated and reviewed in these columns by request. Address requests to Leo L. Beranek, Cruft Laboratory, Harvard University, Cambridge, Massachusetts.

A New Optical Method of Measurement for Phonograph Recordings. GERHARD BUCHMANN AND ERWIN MEYER, E. N. T. **7**, 147–152 (1930). Because of the rapid development in recent years of phonograph recording techniques, attention has been directed to methods of measurement in this field. Modern electrical recorders consist of three fundamental units: the microphone, the amplifier, and the electromagnetic cutting head. Methods of determining the characteristics of microphones and amplifiers are well known. But so far simple methods for determining adequately the characteristics of lateral-cut recording heads have not been worked out. In order to measure the frequency response of a cutter, a constant alternating-current potential, or at least a known voltage, must be applied across the head at all frequencies in the recording range and the recorded amplitude for the various frequencies must be measured. Furthermore, a record having known amplitudes at various frequencies is necessary in order to determine the frequency-response characteristics of electrical or mechanical phonograph reproducers. Thus, the calibration of both recording and reproducing equipment is dependent upon the amplitudes of recorded grooves on phonograph disks.

The oldest and most widely used method of measurement is the microscopic one. It gives the amplitude directly. However, there is considerable difficulty with this type of measurement, especially at the high frequencies, because of the minute amplitudes. The most important magnitude in phonographic recording and reproduction is not the amplitude itself, but the stylus-point velocity; i.e., the product of amplitude and frequency. An ideal electrical cutter should record constant stylus-point velocity at all frequencies when activated by an alternating current of constant voltage; amplitude, on the other hand, varies inversely with frequency for constant voltage input.

A second measuring method which one of us* developed recently employs an electrical pick-up, the output of which is a measure of the recorded amplitudes. Frequency characteristics of the pick-up are eliminated by so controlling the speed of the reproduction turntable that the same frequency is always reproduced for measurement regardless of what the original recorded frequency was. The chief disadvantage of this method is the amount of apparatus—amplifier, rectifier, variable-frequency generator for the comparison tone—necessary to carry it out. The method yields amplitudes, not in their actual magnitudes but in the magnitudes traced by the pick-up needle. That is, the magnitudes are those which hold for actual reproduction. More will be said of this later.

A very simple method will now be described by means of which stylus-point velocity values may be read directly in absolute units from phonograph records. The method involves nothing more than a quantitative evaluation of the optical patterns which are reflected from a record. The theoretical development is as follows. Let a sinusoidal wave of given amplitude and frequency be cut on a record. Curve AE of Fig. 1 represents a short segment (one wavelength) of a side wall of such an engraved groove. Let parallel light fall on this segment. The angle which a reflected ray forms with an incident ray is dependent upon the point at which the reflection takes place. It varies from zero at the points A, C, and E to a limiting angle γ at the points of change, B and D, which are the points of intersection of the sinusoidal curve with its axis $o-o$. The important thing is that the magnitude of the angle γ is dependent upon the slope of the curve at this point, that is,

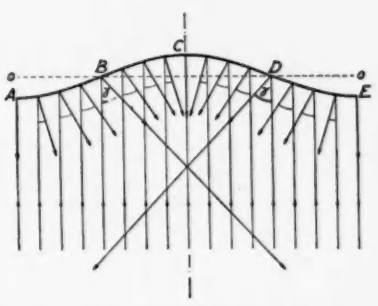

FIG. 1. Reflection of parallel light from a sine wave.

upon the differential quotient, which in turn is dependent upon the product of the angular velocity of the stylus and the amplitude. Figure 2 illustrates a condition where several wave-lengths are observed. Again parallel rays of

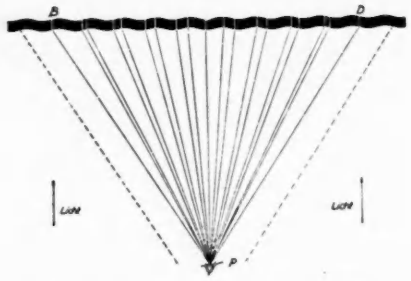

FIG. 2. Points of reflection from several wave-lengths in a sinusoidal wave chain.

light strike the reflecting wall of the groove. An observer at the point P sees a series of light points. The two most widely separated points B and D, points at which the light reflection is greatest, lie at points of change on the sinusoidal curve and are therefore determined by the magnitude of the stylus-point velocity. Points of reflection beyond B and D which could contribute to the image do not exist for an observer at the point P. Movement of the wave—rotation of the phonograph record—causes the reflecting points to move and gives the impression of a band of light extending from B to D. The width of this light band observed at a given point, P, is a measure of the stylus-point velocity. Figure 3 shows this light phenomenon on a wax record on which a constant frequency (600∼) and constant amplitude has been cut. The beginning and end of the record have a few unmodulated grooves. The sun served as a light source and the record was tilted so as to give maximum effect for photography. The record was rotated so that the single reflections would fuse. It may be seen that the light band is sharply delineated and can easily be measured.

It seems remarkable that the light band has a constant width across the record despite the differences in arc of successive grooves. So far it has been assumed that the sinusoidal wave was cut along a straight line instead of along a circular arc as is actually the case. If the groove shown in Fig. 2 were cut on a circle with the convex side toward P,† the angle of the reflected ray with the incident ray at points B and D would be less and the light-band width would be less. Consequently, it appears that the light-band width is proportional to the radius of the circle constituting the axis of the waves. This effect is offset by the fact that the wave-length of a given recorded frequency decreases with the radius of the circle on which it is recorded. This decrease in wave-length is accompanied by an increase in the slope of the curve at the points where it crosses the axis, an increase which is inversely proportional to the radius. Thus the light band of a constant stylus-point velocity recording is constant in width across the record.

Figure 4 shows a tone of 435∼ at different intensity levels. Amplitudes were varied as 1 : 2 : 4 : 8 : 16. Light bands in this picture are reflected from a standard commercial pressing. The source of the parallel light beam was a Wolfram point lamp. The two oblique bands are due to the diffuse daylight entering the room from two windows. As was mentioned above, parallel rays of light are necessary to bring out sharp delineation of the light bands. If a good source of parallel light is used, even very small amplitudes and amplitude differences may be measured. The sharpness of the bands is limited not only by the kind of source used, but also by the fact that the groove walls do not reflect purely geometrically, but somewhat diffusely. This is shown by the fact that an unmodulated groove (see Fig. 1) does not reflect an infinitely fine line even when illuminated by strictly parallel light rays. The light-band width of an unmodulated groove is dependent upon the coarseness of the recording material and the cleanness of the cut. It may be considered as a measure of the surface noise level of the disk.

For modulated grooves, the light-band widths are a measure of stylus-point velocity. In order to determine the characteristic of a cutter, a constant a.c. voltage is maintained across the head while the frequency is varied

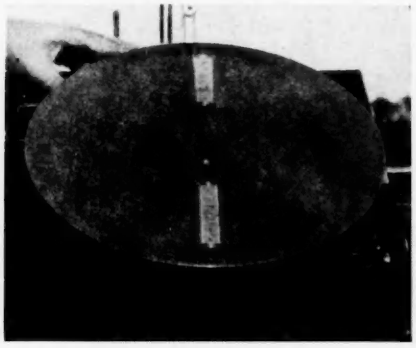

FIG. 3. Wax record of constant frequency and amplitude.

FIG. 4. 435∼ tone with different amplitudes.

throughout the range to be tested. The light-band pattern gives the response curve directly.

The absolute magnitude of stylus-point velocity can be obtained from the light-band width by a few simple measurements. Figure 5 explains this process. ABC is a sine wave along a circle with the radius R, α the amplitude and ω the angular velocity of the cutting stylus ($\omega=2\pi f$). M is the center of the phonograph record. Let parallel light rays strike the groove. Let the distance of an observer be great in comparison to the band width b. Then the reflected rays can be assumed to be parallel. From Fig. 5 the relation between b and R is

$$\sin \alpha = b/2R, \tag{1}$$

where α is half the angle at M. Also, since α is the slope of the sine wave at the point of intersection with the axis, it follows that

$$\tan \alpha = \alpha\omega/2\pi Ru \tag{2}$$

where u is the number of revolutions per second of the phonograph record—normally 78/60. The quotient on the

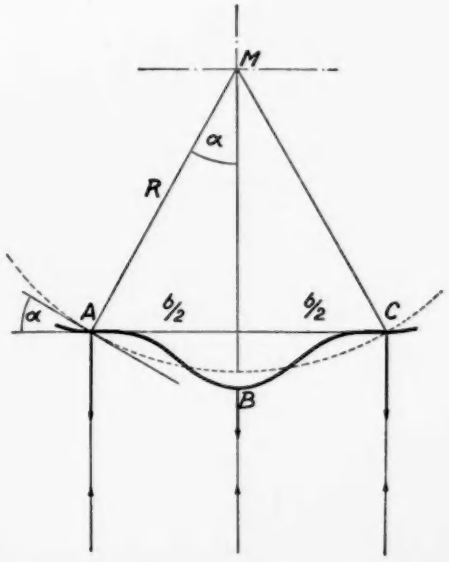

FIG. 5. Absolute measurement of stylus-point velocity.

right side of Eq. (2) is the ratio of two velocities, the radial and the tangential cutting velocity. From (1) and (2) it follows that the stylus-point velocity is

$$\alpha\omega = b\pi u/\cos \alpha, \tag{3}$$

and since the band width is usually small in comparison with the radius, $\cos \alpha$ can, in most instances, be set equal to 1. If the rotational speed u is known and the band width b is measured, for example with a telescope and scale, the stylus-point velocity $\alpha\omega$ can be determined. If $2\pi f$ (where f is expressed in cycles per second) is substituted for the angular velocity of the stylus point ω, and the assumption $\cos \alpha = 1$ is made, the following equation for the amplitude results

$$\alpha = bu/2f. \tag{4}$$

In the development of (3) and (4) it is tacitly assumed that the reflecting groove wall is perpendicular to the plane of the drawing in Fig. 5. This is not the case in reality since the side walls are oblique to the plane surface of the record in a manner determined by the face angle of the cutting stylus. The plane of observation and the plane of the record surface are not the same. A geometric analysis shows, however, that this fact does not affect the results. It should be mentioned that various recording firms use different face angles on their cutting styli and, consequently, each record must be adjusted to the incident light so that the optimum light band patterning is achieved.

Equation (4) was experimentally checked for rather large amplitudes by direct microscopic measurement (see above) and the microscopic and light-band width measurements agreed within a few percent.

This method of measurement also permits interesting interpretations of the distortion factor in recorders. The frequency run in Fig. 6 shows, for example, that at large amplitudes the cutter produces rather strong nonlinear distortion, i.e., harmonics are generated from a sinusoidal input. This is especially noticeable at the low frequencies where a fine bright line within the light band may be observed which is similar in pattern to that noticed on an oscillogram (especially when the waves are very short in length and of high amplitude) where harmonics are present in an almost sinusoidal wave. Furthermore, the harmonics from the frequencies 800, 1600 and 3200~ are accentuated by the peak of this cutter at 6400~. The distortion at 3200~ is great and may readily be seen on the record.

In conclusion, a comparison of other methods with the one described here may be made. The microscopic measurement yields the recorded amplitude directly; the new method yields stylus-point velocity directly. Equations (3) and (4) show that the two procedures are directly comparable. The electrical procedure mentioned in the

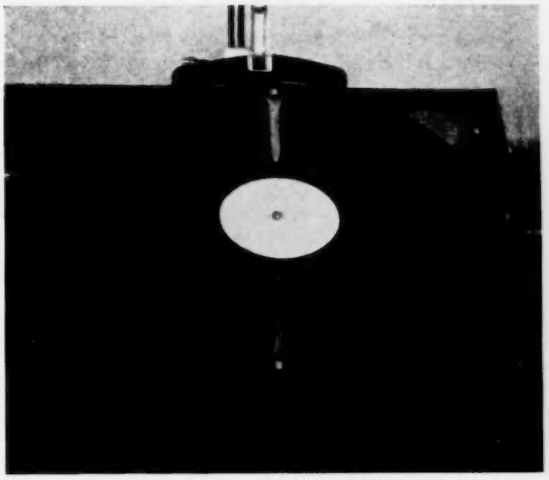

FIG. 6. Frequency response characteristics of a cutting head.

beginning yields slightly different quantities. Instead of the actual recorded amplitude it yields the amplitude traced by the pick-up needle-point.

J. M. COWAN

* Erwin Meyer and Paul Just, "Frequenzkurven von elektrischen Tonabnehmern und mechanischen Grammophonen." E. N. T. **6**, 264 (1929).

† In this case, at a very short distance from the record, the width of the light band would be independent of the position of the point P.

Investigations of Vocal Cord Vibration. W. TRENDELENBURG AND H. WULLSTEIN, Sitz. Preuss. Akad. Wiss. Phys.-Math. Klasse **21**, 399–426 (1935). The purpose of this investigation was to determine the form of vibratory motion of the vocal cords; further, to determine whether or not this form is altered by changes in the supralaryngeal cavities.

A review of the literature showed that most workers had not touched the problem, or had assumed that cavity changes alter the form of the cord vibration. No direct evidence bearing on the problem was found.

Fresh human and calf larynx preparations were used. An arrangement was provided so that a controllable and measurable stream of air could be forced up through the trachea, thus bringing the cords into motion. The preparations were imbedded in Plastellin and provision was made so that Plastellin forms approximating the shapes of the cavities for the vowels "*a*" and "*o*" could be attached above the preparations.

Two means for recording movements of the cords were provided. First, the cords were made to function as a condenser microphone. The fixed plate, a small, round copper disk, was placed about 6 mm below the lower surface of the cords. The cords themselves constituted the moving plate, and leads were inserted in them at the anterior juncture point and in the posterior end of each cord. This "microphone" was fed through an amplifier into an oscillograph. The second means of recording consisted of an optical system arranged so that a beam of light shone up through the trachea and was brought to focus as a narrow slit across the cords perpendicular to their length. The light slit was usually focused across the midpoint of the length of the cords. When the cords were apart, an enlarged image of this slit was focused on moving film. Further, air vibrations either above or below the cords were recorded by means of condenser microphone and oscillograph. Provision was made so that any combination of these various recordings could be made simultaneously on the same film.

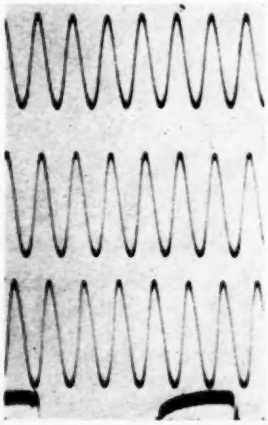

FIG. 1.

Figure 1 shows recordings obtained by the condenser method of tracing the cord movements (human larynx, $f=550\sim$). The top curve was taken without cavities above the preparation, the middle with an "*a*" cavity attached, and the lower with an "*o*" cavity attached. The form of the movement remains unchanged with or without cavities. Recordings of the air vibrations under these conditions showed characteristic differences for the three conditions. When the cavities were attached, perceptually satisfactory vowels were obtained. The form of the tracings is essentially sinusoidal and shows none of the higher partials which were present in the tracings of the air vibrations; the former agree with the latter only in the frequency of the fundamental.

Figure 2 shows shadowgraphs of the movements of the vocal lips at the point where the light slit crossed the cords

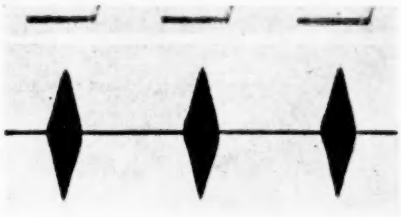

FIG. 2.

(calf larynx, $f=100\sim$). The shadow above the axis shows the movement of one lip, that below, the movement of the other. This type of recording shows a zero phase corresponding to closure of the lips which cannot be detected on tracings by the condenser method. The authors explain the difference by assuming that motion of the cords does not cease with contact of the lips in the closure phase of the cycle, but continues as a pressing together of the lips, followed by a release before the actual opening of the glottis begins. The form of the shadowgraphs is straight lined, analogous to the saw-tooth form of recordings of the motion at a given point on a bowed string.

Figure 3 shows simultaneous recordings of glottal shadowgraphs and air vibrations above a calf larynx preparation ($f=90\sim$). Each period of the air vibration shows two groups of strong high partials (around 4000~). The first of these groups coincides with the glottal opening and the second follows immediately after the glottal closure as shown by the shadowgraphs.

The authors conclude that the motion of the cords as a whole, including motion during the closure phase of the glottal cycle, is essentially sinusoidal. This motion corresponds only to that of the fundamental of the air vibrations, the latter containing strong high partials not present in the cord motion. Alterations of the supralaryngeal cavities do not affect cord motion. The motion of the

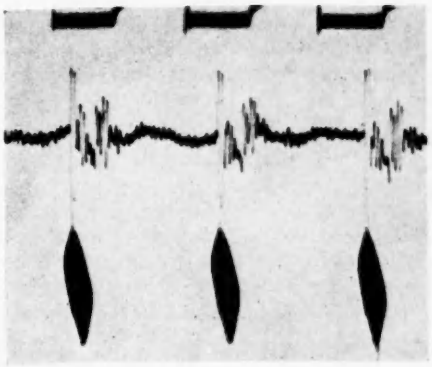

FIG. 3.

glottal opening is similar to the motion at a given point on a bowed string, especially when the glottal closure phase is very short. The amplitude maxima of the high partials in the air vibrations correspond temporally to the opening phase of the glottal cycle. They find the vocal mechanism analogous to that of a stringed instrument. The cord-airstream system during vocalization corresponds to the string-bow system during bowing. Both are undamped systems. The coupled system of supralaryngeal cavities determines the form of the radiated air vibrations just as the coupled system of the body of the instrument does for the stringed instrument. These two latter systems are highly damped. In neither case do changes in the damped systems affect the undamped systems by back-coupling.

J. M. COWAN

Investigation of "Block" Flutes. A. VON LÜPKE, Akustische Zeits. **5**, 39 (1940).—*Translator's note: The block flute is a sort of a fife and is unknown to me. Those who manufacture musical instruments will undoubtedly recognize it and call it by its proper name. The author (v. Lüpke) claims that these instruments are commonly used in the production of school and folk music in Germany and also in the production of chamber music. His method of analyzing the tones is probably of interest. The second harmonic content of typical flutes is shown in the review. The original article shows the relative amplitudes of the third, fourth, and fifth harmonics as well.*

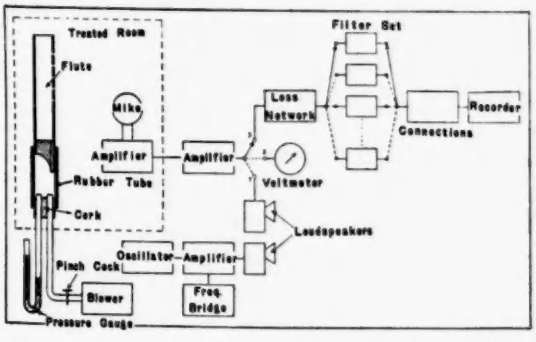

FIG. 1. Arrangement of apparatus for recording the tone-spectrum and for measuring the wind pressure and sound pressure. Switch positions: 1. Determination of pitch; 2. sound pressure measurements; 3. harmonic analysis.

A diagram of the apparatus for analyzing the tone of the block flute is shown in Fig. 1. The flute is played in an acoustically treated room so as to eliminate standing waves. The mouthpiece of the flute is held by a rubber hose to simulate the elastic grip of the hand. A blower is provided whose output is controlled by a pinch cock and whose pressure is measured by a water manometer. The microphone was placed 1.2 m distant from the center of the flute. The output of the microphone was amplified and conducted successively through each of a group of filter sets which enabled each harmonic to be segregated and its amplitude determined. Since the harmonics were not always in the center of the pass bands of the filters the insertion loss of a filter was different for each harmonic. This difference was equalized by a loss network. The apparatus was then so arranged that a synchronous motor switched from one filter to the next successively and automatically inserted the proper loss network. The output of the filters was recorded on a Neumann high speed level

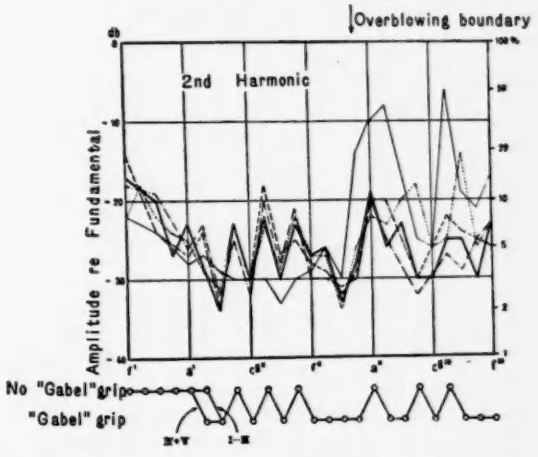

FIG. 2. Relative amplitudes of second harmonics for five old flutes in *f'*.

recorder. This means that the fundamental and each harmonic were recorded side by side on the waxed record on a decibel scale. The gain of the amplifier was always adjusted so that full scale indication on the record was obtained for the fundamental.

A question naturally arose as to what wind pressure should be used for blowing the flute. The author decided that each tone should be blown at that wind pressure which gave the instrument its advertised pitch. A standard signal was produced by the apparatus shown in lower Fig. 1 and the wind pressure was adjusted until zero beat was obtained between the two loudspeakers with the switch in position 1.

Every playable note in two octaves was tested for each of five flutes. The author draws the following conclusions: (1) In comparison to almost all other instruments the tone is lacking in harmonics. (2) The third harmonic is in general stronger than the second. This occurs because of the conical bore of the instrument. (3) For certain notes the harmonics have unstable amplitudes, thus imparting to the tone a certain roughness. (4) The number and

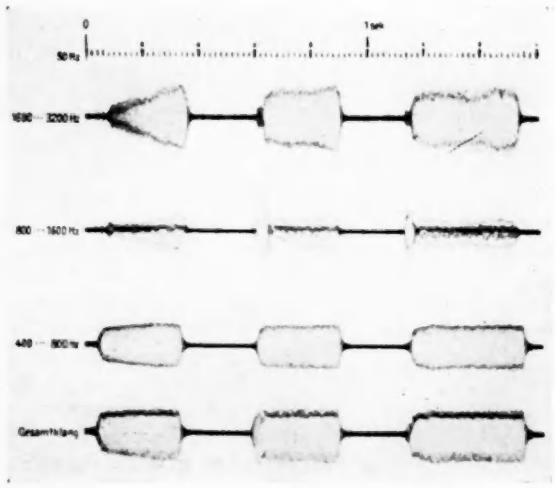

FIG. 3. Transient response of the "Meisterflute" instrument.

strength of the overtones decreases as one progresses up the scale to a certain limit at which the flute becomes "over-blown." At this point the harmonics suddenly become strong and remain so for the remainder of the scale. (5) In Fig. 2, which shows the amplitude of the second harmonic relative to the fundamental as a function of frequence, the similarity of the zigzag portion between a' and a'' for the five flutes is surprising. Beneath the graph is a chart showing the cases for which one or more holes are closed beneath an open hole; this condition being called a "gabel" grip. This zigzag portion of the curve is not in evidence on the graphs for the higher harmonics.

Wind pressure curves were taken for all instruments. The maximum range of wind pressures necessary over the scale was about 1 : 6. This is small compared to the changes encountered in an organ. Also for the block flute the highest wind pressures were necessary at the higher notes, which is the opposite demanded for organ pipes. The curves are more nearly flat for better instruments. The overblowing boundary is distinctly seen on the wind pressure curves.

The transient characteristics of the flutes were tested by recording the outputs of the microphones on a photographic plate, using a three string oscillograph connected to the outputs of parallel band-pass filter sections. The instruments were blown by a recognized artist. A sample record is shown in Fig. 3 for a soprano flute. The growth of the fundamental and of the higher harmonics is clearly seen. In the 800–1600 cycle region certain transient notes occur at the onset of the note, but quickly die out.—L. BERANEK.

A Method for Producing Extremely Strong Standing Sound Waves in Air. HERMANN OBERST, Akustische Zeits. **5**, 27 (1940).—This paper presents a method for producing sound waves of extremely large amplitudes with relatively pure wave forms. Waves of extremely large amplitude are defined as those for which the amplitudes of excess pressure are of the order of the steady pressure in air, i.e., those whose amplitude velocity is comparable with that of the velocity of sound.

The fundamental apparatus is shown diagrammatically in Fig. 1. Two tubes of radii r_1 and r_2 and length l_1 and l_2 are joined together as shown. At the open end of the first tube there is placed a source of alternating sound pressure P_e. Large sound pressures are observed at the closed end of the second tube ($x_2=l_2$) whenever the two tubes are adjusted to resonance. A brief approximate theoretical treatment follows.

If one produces plane standing sound waves in a tube of length l_1, rigidly terminated at one end and open at the other, at a frequency f whose wave-length is λ, then pressure maxima will occur whenever

$$l_1=\lambda\cdot(2n+1)/4, \text{ where } n=0, 1, 2, 3\cdots. \quad (1)$$

At $x=l_1$, for the special case of small damping at the side walls of the tube, the magnitude of a pressure maximum is

$$|p_1(l_1)_{\max}|=P_e/\beta_1 l_1, \quad (2)$$

where β_1 is the attenuation constant per centimeter length of tube for a plane wave in the tube at the frequency f and P_e is the excess pressure at the open end. If a second tube of length l_2 and radius r_2 is joined on at the center of the rigidly terminated end of the preceding tube, and if r_2 is small compared to r_1, then the resonance conditions in the first tube will be essentially unchanged. The excess

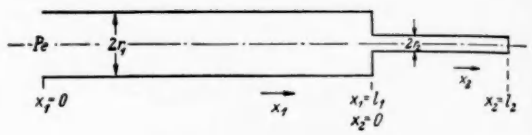

FIG. 1. Diagram of the tube arrangement.

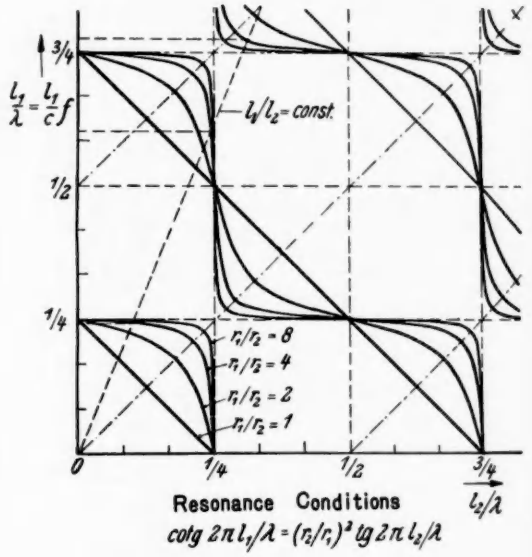

FIG. 2. Resonance conditions for the tube arrangement.

pressure at the open end of this second tube will be equal to $p_1(l_1)$ by the equation of continuity and the excess pressure at $x_2=l_2$ will be given by an expression similar to (2) except with all subscripts changed from one to two

and with $P_e = p_1(l_1)$. We can then write that

$$|p_2(l_2)_{max}| = P_e/(\beta_1 l_1 \beta_2 l_2). \quad (3)$$

In practice one need not necessarily make r_2 small compared with r_1; in fact, because β increases with decreasing tube radius it is desirable not to make r_2 too small. A more accurate treatment of this case for r_1 and r_2 of similar magnitude is given in this paper and leads to an equation which reduces to (3) for $r_1 \gg r_2$. The equation expressing the conditions of resonance for the case of r_1 not large compared to r_2 is of especial interest, namely,

$$\cot(2\pi l_1/\lambda) = (r_2/r_1)^2 \tan(2\pi l_2/\lambda). \quad (4)$$

A plot determined from this equation is shown in Fig. 2. It is seen that for a constant ratio of r_1/r_2 and of l_1/l_2 (indicated by dotted straight lines intersecting the origin) the resonant frequencies are not related by simple whole numbers. (Note that in using this chart either the ordinate or abscissa is proportional to the driving frequency f.) This means that the normal frequencies are not in harmonic relation and by using two tubes, one smaller than the other, the overtones will not be at resonance when the fundamental is and, hence, at the output of the second tube a nearly pure sinusoidal variation of pressure results.

As a source of sound, the authors used a hole-siren. This consisted of an air blower whose output was a 5-cm opening. At this point the air stream was modulated by a revolving perforated aluminum disk. The holes in the disk were also 5 cm in diameter. The frequency region was limited by the number of holes in the disk and the blower speed, the upper limit being 200 cycles.

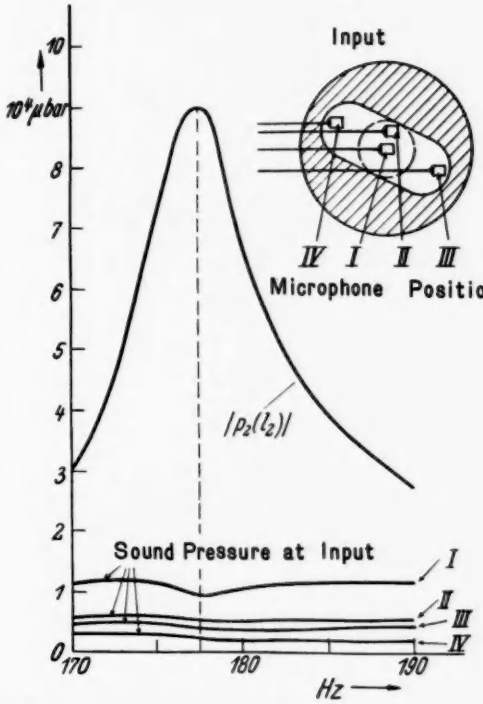

FIG. 3. Sound pressures at different points in the tube arrangement.

The dimensions of the pipes employed were $l_1 = 107$ cm, $l_2 = 45$ cm, $r_1 = 6$ cm, $r_2 = 1.25$ cm. The sound pressure at the end of the tube $x_2 = l_2$ was calculated from the sound radiation pressure which was measured with a water-filled U manometer. In the vicinity of the velocity node, $x_1 \doteq \lambda/4$, a very light, thin rubber membrane was stretched, distorting the sound wave as little as possible. This served as a filter to remove the steady air pressure of the blower, but allowed the alternating sound pressures to pass into the tube system. A small jet was placed at $x_1 = l_1$ to equalize small steady pressures produced by stretching of the membrane. After these precautions the U manometer indicated properly the sound radiation pressure at $x_2 = l_2$.

The resonant frequencies of the tube were in good agreement with those predicted by the theory (Eq. (4)), using the dimensions mentioned above.

The author spends considerable time in checking the exact theory against the measured results. In the mathematics it is assumed that the excess pressure at $x_1 = 0$ is constant as a function of frequency and this was checked experimentally as shown in Fig. 3. The excess pressure at $x_2 = l_2$ is also shown in this figure and reached a maximum of 9×10^4 microbars at resonance. A frequency analysis at the position $x_2 = l_2$ at the resonant peak shown in Fig. 3 indicated a distortion of about 5 percent, while an analysis of the pressure P_e indicated a total distortion of about 30 percent. This is as expected from the discussion above. Lower harmonic distortions and higher pressures might be obtained, using an electrodynamic driving unit as a source in place of the hole-siren.

For the maximum pressure at 180 c.p.s. shown in Fig. 3 a particle amplitude of about 2 cm exists at $x_2 = 0$.

The method is applicable for studying stream-phenomena in strong sound fields. Dust and smoke particles form into stria at the velocity loop, making it possible to study the air-streaming around larger bodies such as a Rayleigh disk. The apparatus is suitable for testing condenser and crystal microphones in large sound fields. It is also useful for demonstration purposes, especially in making visible the condensations and rarefactions of the air at the pressure loop by means of stroboscopic illumination of the rubber membrane.—L. BERANEK.

The Recording of Small Pitch-Variations. M. GRÜTZMACHER AND W. LOTTERMOSER, Akustische Zeits. **5**, 1 (1940).—The authors, in two previous works,[1] have described a pitch recorder to be used over a range of one to two octaves with an accuracy of three to six percent. The new apparatus described here is useful for recording very small variations in pitch.

Figure 1 illustrates the successive transformations which the received wave whose pitch is to be determined undergoes. First, all harmonics of the fundamental are removed from the electrical output of the recording microphone by a filter. This leaves a current whose wave form is, for example, of the type shown in Fig. 1(a). Then, by means of apparatus not described in this paper, this current is transformed into a succession of voltage peaks as shown in (b) spaced along the time axis with the instantaneous period of the current of (a). (Hereafter these peaks are

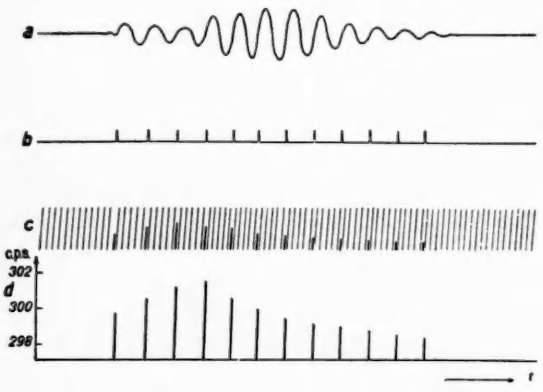

FIG. 1. The successive transformations leading to the recording of small pitch variations.

designated as "control peaks.") Next, a high frequency saw-tooth wave generator is provided whose frequency is known and maintained constant by some external method. The control peaks are then impressed on this saw-tooth generator such that each time a control peak in (b) occurs the increasing current of a saw-tooth wave is suddenly reduced to zero. For an applied tone of constant frequency the control peaks will be evenly spaced in time and those "chopped-off" segments of the saw-tooth wave will have identical lengths. If, however, the period of the applied tone is varying, then the "chopped-off" segments will be of different lengths as seen in Fig. 1(c). Finally, a means is provided to allow observation of only those saw-tooth wave portions which are interrupted by the control peaks. The final record appears as shown in (d). Obviously the accuracy of the method can be increased by employing saw-tooth waves of higher frequency.

A schematic diagram of the apparatus is shown in Fig. 2. This diagram shows the method used to separate the "chopped-off" segment from the other waves. The output of the upper saw-tooth (Kipps) wave generator is connected to one pair of plates on a cathode-ray tube. The second pair of plates is connected to the output of a low-frequency saw-tooth wave generator whose period is determined by the control peaks. This generator causes the chopped-off stroke to appear always at the same portion of the tube's screen. If a mask with a slit is provided, a photographic film may be used to record the successive strokes of varying lengths that occur.

The authors are careful to mention that one cannot speak of a frequency variation of a *sinusoidal* voltage. That is, a variation in wave form necessarily means the existence of other frequencies. In the method described here the periods of the null or axis-crossing points of the voltage wave are measured and one says that this is a measure of the instantaneous frequency of the voltage under investigation. The apparatus measured pitch with a precision of better than one percent for a 40-decibel variation in amplitude of the applied tone.

A number of typical analyses are shown in the paper. The frequency of an oscillator was found to be very constant over the measured period of several seconds. This

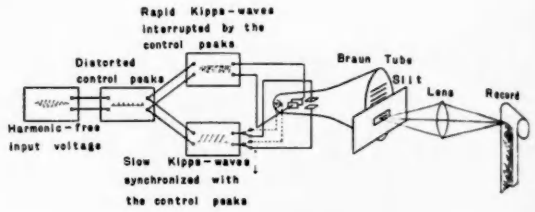

FIG. 2. Arrangement of apparatus.

oscillator was then used in the recording of a phonograph record. The record was played back and the output of the pick-up analyzed. Rapid variations of about 1 cycle at 200 cycles were recorded and were found to be caused by a resonance in the pick-up arm. A similar investigation was made of a sound-on-film recording. Especially interesting are analyses made of pitch variations in the output of a violin under different bowing conditions. For medium bow pressures, very little variation in pitch was observed during the duration of the note. For heavy bow-pressures variations of 1 to 1.5 percent from the mean were found. Artists' vibratos on the violin were also recorded. Larger frequency variations were measured for wind instruments, *ca.* 2.5 percent. The flute showed pitch variations of as high as 4 percent. The greatest variation was found for the human singing voice. For this, pitch variations as large as 6 percent about the mean were recorded.—L. BERANEK.

[1] M. Grützmacher and W. Lottermoser, Akustische Zeits. **2**, 242 (1937); **3**, 193 (1938).

A Method for Determining Acoustic Reflection Coefficients of the Earth's Surface. W. ERNSTHAUSEN AND W. V. WITTERN, Akustische Zeits. **4**, 353 (1939).—Because of the increased use of echo-sounding in aviation, the authors believed it of interest to investigate the reflecting efficiency of different types of earth's surfaces over a reasonably wide frequency range. They investigated the absorption of sound at normal incidence to different earth's surfaces with and without coverings such as sand and snow.

To perform the measurements a loudspeaker was suspended from a boom about 20 meters above the surface

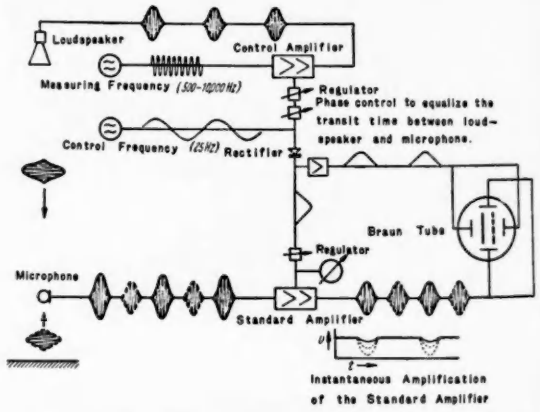

FIG. 1. Diagrammatic representation of the method of measurement.

under test. A microphone was placed directly beneath the loudspeaker and about 2 or 2.5 meters above the test surface. The sound proceeded from the loudspeaker, passed the microphone, reflected from the test surface, and again passed the microphone. Assuming that the microphone was equally sensitive from both sides it was only necessary to compare at the microphone the amplitude of the sound before reflection to that after reflection. To this end, the sound was broken up into a number of pulses of short duration and the amplitudes of two successive impulses, direct and reflected, at the microphone were compared to give the reflection coefficient.

Figure 1 shows a block diagram of the apparatus. The measuring frequency is generated by an oscillator and

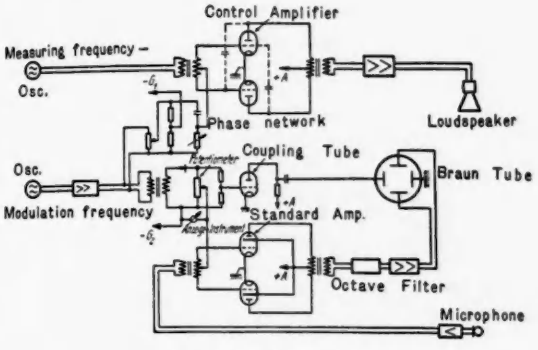

FIG. 2. Wiring diagram.

passes to a modulator or "control amplifier." The modulation or control frequency of 25 c.p.s. is used to break up the higher frequency into a series of impulses occurring once every $\frac{1}{25}$th of a second. The 25 c.p.s. control frequency is, in addition, passed through a rectifier and these rectified impulses are used to control the amplification of the "standard amplifier." The two voltage impulses, produced by the direct and reflected sound waves, pass from the microphone to the input of the standard amplifier. The instantaneous amplification of the standard amplifier, as shown in the lower right-hand diagram, is varied by the d.c. pulses from the rectifier. The magnitude of the variation in gain can be changed by adjusting the lower regulator. By this means, the standard amplifier possesses the highest gain for the reflected signals, while its gain is diminished for the primary signals. Obviously, a phase control must be provided to atone for the transit time between loudspeaker and microphone. In practice, the

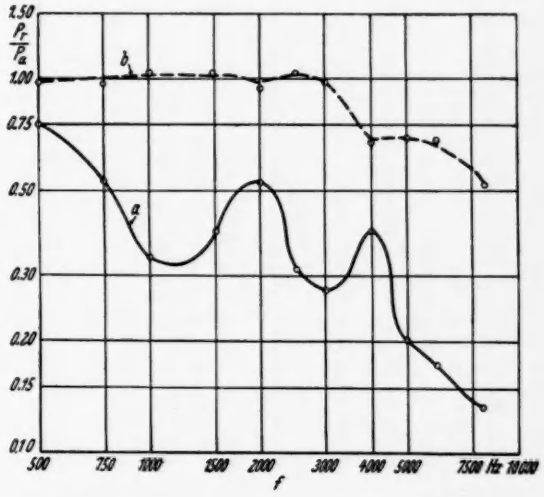

FIG. 3. Reflection from concrete (a) with 6 cm of snow; (b) without snow.

lower regulator (Fig. 1) was adjusted until the impulses as viewed on a cathode-ray tube were of the same length. The magnitude of the d.c. pulses was read on an indicating instrument at the standard amplifier and related to the reflection coefficient of the surface under test. In order to avoid overlapping of the two lines on the cathode-ray tube, the d.c. pulses were also used to change the bias on the horizontal plates of the tube each time a primary pulse was being observed. A schematic diagram of the apparatus is shown in Fig. 2 and needs no further description.

Reflection coefficients were measured for concrete surfaces with and without various thicknesses of sand and for rigid surfaces covered with either gravel or snow. Measurements were also performed to determine the reflection from a wavy or other irregular surface and for reflection from circular plates and rigid cylinders. Experimental data for all of these are plotted in the paper. In general it was found that except for a few things such as sand and snow, the reflecting efficiency of most common earth's surfaces was high. For uneven sand-covered surfaces the absorption became very large at higher frequencies. Figure 3(a) shows a typical curve of pressure reflection coefficients for a surface covered with a six-centimeter layer of fine texture snow. The absorption increases with frequency. Figure 3(b) shows the absorption of the surface when not covered with the snow. The density of the snow determines to a large extent its absorbing properties.—L. BERANEK.

Book Reviews

Elements of Acoustical Engineering. HARRY F. OLSON. Pp. 344. D. Van Nostrand Company, Inc., New York. Price $6.00.

This treatise in the rapidly advancing field of acoustical engineering is very definitely needed. The author, Dr. Harry F. Olson, as a student of theoretical physics and as a successful creative acoustical engineer, writes from a point of vantage which is clearly in evidence. He can and does put into this book what every engineer and physicist who is in contact with acoustics should know. To the engineer he gives in brief form the theoretical bearing of virtually every practical application in acoustics, and to the physicist there are presented many practical facts with which he should be acquainted. Dr. Olson's new work is not a textbook in the ordinary sense, though the subject matter is that given in thirty lectures at Columbia University. For the same reason that these lectures doubtless were quite well understood by, and valuable to, Dr. Olson's audience at Columbia, this book could wisely be used as a text by a teacher who is also a thorough student of the theory of acoustics. The book is certainly a very valuable reference work and is quite readable beginning at any subject, if one is familiar with the first chapter.

Although the author says in the preface that, "No special knowledge of acoustic principles is required for an understanding of the subject matter of the book," this statement may need some interpretation. It is the purpose of the author to outline the essentials from the applied standpoint, and, to this end, mathematical derivations are to be taken for granted. A reader desiring applied acoustics must therefore omit any troublesome mathematical procedure which is suggested and focus his attention chiefly upon important conclusions. Yet no engineer should conclude that, without a thorough-going theoretical knowledge of acoustics, he can hope to do creative work with ease in the field of applied acoustics.

Throughout the text explanations are given in terms of electric circuit theory, and wisely so. But since the analogies between acoustic circuits and electric circuits are not at all physical but entirely mathematical, he who would think constructively in acoustics has a great advantage if, to him, the physics of acoustics is very clear. The book is amply illustrated with important diagrams and plotted data. It contains a great amount of material presented in a convenient and compact form. The book itself is altogether pleasing. It should be regarded as an essential part of the working library of every engineer and physicist who is interested in acoustical engineering.

G. W. STEWART
State University of Iowa

Einfuhrung in die Akustik (Introduction to Acoustics). FERDINAND TRENDELENBERG. 277 pages and 215 figures. Julius Springer, Berlin, Germany. (In German).

This book is a well-organized and useful, though not advanced, textbook of the science of acoustics. The illustrations are good and there are many of them. The author begins by devoting many pages to a discussion of periodic phenomena. He covers Fourier analysis, Lissajous patterns, free and forced oscillations and coupled circuits. He then derives in simple fashion the wave equation and discusses the normal modes of vibration of air-chambers, strings, membranes, etc. The second section is devoted to a discussion of the different ways that one might measure the strength of a sound field. Section three is devoted to all types of sound producers, including mechanical devices, musical instruments, the human voice, and electroacoustic sound generators. He also gives the familiar theoretical treatment of radiation from a sphere and from a rigid piston. The fourth section is devoted to a discussion of sound propagation including a graphical explanation of Huygens principle, reflection, diffraction and refraction. He also brings into this section a discussion of resonators and acoustical filters and a short section on room acoustics. The fifth section deals with sound detection and sound recording. The different types of microphones are briefly described and some methods of absolute calibration are mentioned. The ear as a detector of sounds is dealt with more completely than are electrical devices. He very briefly discusses the common methods of sound recording. The final section is devoted to a discussion of sound analysis and the physical characteristics of the human voice and musical instruments.

The chief criticism of this textbook is that because of the large number of subjects covered it is impossible to treat any one of them thoroughly. In many cases the author gives only a brief description of the working principles of a device whereas a mathematical treatment would be necessary to understand it properly. However, he has carefully given extensive references in the form of footnotes to which the reader may refer for more complete treatments. These references are largely in German, although he does refer extensively to the works of Fletcher and to the American contributions to room acoustics. The author has added a useful appendix giving a list of definitions of thirty-one common acoustical expressions and a summary of the most important acoustical formulas used in practice. Considerable handbook material is included such as sound velocities in different media and acoustical absorption coefficients. He gives useful tables of the frequencies of the intervals of a twelve-step tempered musical scale for two different reference pitches.

LEO L. BERANEK
Harvard University

References to Contemporary Papers on Acoustics

MOST of the titles of papers appearing in other languages than English have been translated; Abstracts in English of many foreign papers have appeared or soon will appear in Science Abstracts, Section A. Where references are made to Science Abstracts, the reference is to the volume number and abstract number. The abbreviations of the names of journals are those used in Science Abstracts and can be found in any annual index to those abstracts.

The numbers appearing at the head of each entry are the numbers which identify the subject headings in the Cumulative Index of this journal issued in November, 1939. This same Classification of Subjects can be found in the index to Volume 11.

Compiled with the generous assistance of F. E. Orton, Physics Librarian, University of Michigan.

2. Architectural Acoustics

2.4 **Form and Propagation of Sound Waves in a Space Limited by Absorbing Surfaces.** J. Brillouin. J. de Physique et le Radium **10,** 497–503 (1939). Sci. Abs. **A43,** 1230 (1940).

2.4 **The Absorption of Sound by Systems with Double Resonance.** V. S. Nesterov. J. Techn. Physics, U. S. S. R. **9,** 1727–1739 (1939). (In Russian.)

2.5 **Effect of Paint on the Sound Absorption of Acoustic Materials.** V. L. Chrisler. Bureau of Standards, J. of Research **24,** 547–553 (1940).

2.5 **Sound-Absorbing Properties of Common Outdoor Materials.** G. W. C. Kaye and E. J. Evans. Phys. Soc., Proc. **52,** 371–379 (1940). Sci. Abs. **A43,** 1903 (1940).

2.7 **The Effective Values of Reverberation.** A. M. Kostzov. J. Techn. Physics, U. S. S. R. **9,** 919–929 (no date given). Wireless Engineer **17,** Abs. 1466 (April 1940).

2.8 **Simple Apparatus for the Measurement of Reverberation Time.** A. Calza and M. Nuovo. Ricerca Scientifica **10,** 1090–1095 (1939). Sci. Abs. **A43,** 1240 (1940).

2.8 **Measurement of Reverberation Time with Several Microphones.** F. Verchiacchi and M. Nuovo. Ricerca Scientifica **10,** 1018–1020 (1939). Sci. Abs. **A43,** 1239 (1940).

2.9 **Soundproofing Research; Boeing Development of Acoustical Laboratory in Cooperation with University of Washington Physics Department.** K. Martinez. Aero Digest **36,** 26–28 (1940).

2.9 **Making Walls Sound-Proof; Radio and Moving Picture Studios.** M. Rettinger. Engineering News-Record **124,** 723–725 (1940).

3. Books and Bibliographies

3. **Index to Technical Papers by RCA Engineers. 1919–1939.** 1081 items covering Acoustics, Phonographs, Public Address, Sound Motion Pictures, Tubes, etc. RCA Institute Technical Press.

4. Ear and Hearing

4.1 **Über die Stärke der Vibrationsempfindung und ihre Objektive Messung.** Georg v. Békésy. Akustische Zeits. **5,** 113 (1940).

4.1 **The Residue, a New Component in Subjective Sound Analysis.** J. F. Schouten. K. Ned. Akad. Wet., Proc. **43,** 356–365 (1940). (In English.) Sci. Abs. **A43,** 1901 (1940).

4.1 **Über die Physiologischakustischen Kennzeiten von Aus-Gleichscorgängen.** Wolfgang Türk. Akustische Zeits. **5,** 129 (1940).

4.4 **Partial Deafness and Hearing-Aid Design.** W. B. Beasley. J. Soc. Mot. Pict. Eng. **35,** 59 (1940).

4.5 **Remark on "Inductive Systems for the Deaf."** D. W. Aldous. Wireless World **46,** 172 (1940).

4.5 **Use of "Warblestone" in Precision Audiometry.** E. G. Burr. Roy. Soc. Canada, Trans., Section III, **33,** 73/81 (May, 1939).

4.5 **Aids to Hearing: Some Problems of Design: Methods of Testing, and Tone Control in Hearing Aids.** T. S. Littler. Wireless World **46,** 167–170; 205–206 (1940).

4.5 **A Hearing Aid for Research and Group Use.** E. W. Marchant and T. H. Turney. J. Sci. Inst. **17,** 149–155 (1940).

5. Applied Acoustics, Instruments and Apparatus

5.1 **Radio Progress During 1939; Electroacoustics.** I.R.E., Proc. **28,** 99–102 (1940).

5.1 **Sound.** S. G. Taylor. Communications **20,** 17–23, 23, 27, 32 (1940); Communications **20,** 17–18, 36 (1940).

5.4 **Analysis of Low Frequencies by Light Diffraction at Capillary Waves.** H. E. R. Becker. Ann. d. Physik **36,** 585–608 (1939). Sci. Abs. **A43,** 1243 (1940).

5.4 **Suchtonanalyse mit Mechanischem Bandfilter und Hochliegender Trägerfrequenz. (Analysis with Mechanical Band Pass Filter and High Carrier Frequency.)** Gerhard Buchmann. Akustische Zeits. **5,** 7 (1940).

5.7 **Portable Apparatus for Measurement of Standard Note of Musical Scale.** A. Barone. Ricerca Scientifica **10,** 1021–1024 (1939). Sci. Abs. **A43,** 1244 (1940)

5.7 **Die Aufzeichnung Kleiner Tonhöhenschwankungen. (The Recording of Small Pitch Variations.)** M. Grützmacher und W. Lottermoser. Akustische Zeits. **5,** 1 (1940).

5.8 **Production of the Modern Loudspeaker.** H. GOLDEN. Communications **20,** No. 3, 15–16 and 37 (1940).

5.8 **Vented Speaker Enclosure.** C. E. HOEKSTRA. Electronics **13,** No. 3, 34 and 54, 56 (March, 1940).

5.8 **The Coaxial Loudspeaker.** B. OLNEY. Electronics **13,** No. 4, 32–35 and 106–108 (April, 1940).

5.8 **Acoustic Line Loudspeakers.** W. D. PHELPS. Electronics **13,** No. 3, 30–32 (March, 1940).

5.9 **Prove su laringofoni. (Test of Laryngophone.)** Alta Frequenza **9,** 237 (1940).

5.9 **The Western Electric "Multimike" Microphone with Bi-Directional, Non-Directional, Cardoid, and Three Hypercardoid Patterns by Selector Switch.** Communications **20,** No. 4, 22 (1940).

5.9 **The Laryngophone.** J. DE BOER AND K. DE BOER. Philips Techn. Rev. **5,** 6–14 (1940).

5.9 **Telephony in Noisy Surroundings and Tests on Laryngophones.** A. FERRARI-TONIOLO. Alta Frequenza **9,** 4–24 (1940). Wireless Engineer **17,** Abs. 1897 (1940). (Quite a lengthy abstract.) (See also 5.17.)

5.9 **Electroacoustic Sound-Pressure Measurements in Practice.** GOSEWINKEL AND SPANDÖCH. Archiv für Technisches Messen No. **140,** T19–T20 (February, 1940).

5.10 **Electro-Sonic Altimeter for Aircraft.** J. H. JUPE. Electronics **13,** 54 (January, 1940).

5.10 **A New Application of Echo-Sounding.** C. H. MORTIMER AND E. B. WORTHINGTON. Nature **145,** 212–214 (1940).

5.13 **Safeguarding Theater Sound Equipment with Modern Test Instruments.** GOODMAN, KOWALSKI, HARDMAN, AND STANKO. J. Soc. Mot. Pict. Eng. **34,** 409 (1940).

5.13. **Audience Noise as a Limitation to Permissible Volume Range of Dialog in Sound Motion Pictures.** W. A. MUELLER. J. Soc. Mot. Pict. Eng. **35,** 48 (1940).

5.13 **The Adjustable Equalizer as a Tool for Selecting Best Response Characteristics.** E. S. SEELEY. J. Soc. Mot. Pict. Eng. **34,** 351 (1940).

5.16 **Audition Demonstration.** R. A. CUSHMAN. Bell Lab. Record **18,** 273 (1940).

5.16 **Improvements in Disc Records through Constant Amplitude Recording.** A. W. DUFFIELD. Communications **20,** No. 3, 13–14 and 28 (1940).

5.16 **Index to Technical Papers by RCA Engineers. 1919–1939.** 1081 items covering Acoustics, Phonographs, Public Address, Sound Motion Pictures, Tubes, etc. RCA Institute Technical Press.

5.16 **Stereophonic Reproduction from Film.** HARVEY FLETCHER. Bell Lab. Record **18,** 260 (1940).

5.16 **Stereophonic Reproduction from Film.** H. FLETCHER. J. Soc. Mot. Pict. Eng. **34,** 606 (1940).

5.16 **The Transport of Sound Film in Apparatus for Recording and Reproducing.** J. J. C. HARDENBERG. Philips Techn. Rev. **5,** 74–81 (1940).

5.17 **Receiver Testing with an Artificial Ear.** S. BALLANTINE. Electronics **13,** 34–35 (June, 1940).

5.17 **Telephony in Noisy Surroundings and Test in Laryngophones.** A. FERRARI-TONIOLO. Alta Frequenza **9,** 4–24 (1940). Wireless Engineer **17,** Abs. 1897 (May, 1940). (Quite a lengthy abstract.)

5.17 **Sound Tests of Telephone Ringers and Dials.** N. R. STRYKER. Bell Lab. Record **18,** 343 (1940).

5.19 **A Tuning Fork Maintained by Photocell.** Cambridge Instrument Company. Journ. Sci. Instruments **17,** 72 (1940).

5.51 **Radio Progress During 1939; Electroacoustics.** I.R.E., Proc. **28,** 99–102 (1940).

5.51 **Radio Studio Air Conditioning Plant Approaches Ultimate in Sound Control; NBC Studio in Hollywood.** C. STROCK. Heating and Ventilating **37,** 15–19 (1940).

6. Musical Instruments and Music

6.1 **Portable Apparatus for Measurement of Standard Note of Musical Scale.** A. BARONE. Ricerca Scientifica **10,** 1021–1024 (1939). Sci. Abs. **A43,** 1244 (1940).

6.1 **The Resonoscope (For tuning musical instruments).** S. K. WOLFE AND L. B. HOLMES. J. Soc. Mot. Pict. Eng. **34,** 534 (1940).

6.6 **Dynamics of Pianoforte String and Hammer. Part III. General Theory.** M. GHOEH. Indian Journ. Phys. **13,** 277–291 (1939). Sci. Abs. **A43,** 1233 (1940).

6.7 **Akustische Eigenschaften von Geigen Verschiedener Klangqualität.** HERMANN MEINEL. Akustische Zeits. **5,** 124 (1940).

6.7 **Development of Violin.** R. VERMEULEN. Philips Techn. Rev. **5,** 36–41 (1940). Sci. Abs. **A43,** 1892 (1940).

6.8 **Untersuchungen an Blockflöten. (Researches on the Flute.)** ARNDT VON LÜPKE. Akustische Zeits. **5,** 39 (1940).

6.9 **Electronic Music.** L. E. C. HUGHES. Nature **145,** 170–174 (1940). Sci. Abs. **A43,** 1232 (1940).

7. Noise

7.1 **Noise in Burglary-Resistive Vaults under Normal and Attack Conditions.** H. D. BRAILSFORD. Underwriters' Lab. Bull. Research. No. **17** (July, 1940).

7.7 **Sound Level Ratios Key to Noise Problem.** El. World **113,** 1731 (1940).

7.7 **How to Use Sound-Deadening Materials in Air Conditioning Equipment.** P. H. GEIGER. Heating and Ventilating **37,** 42–45 (1940).

7.7 **Soundproofing Research; Boeing Development of Acoustical Laboratory in Cooperation with University of Washington Physics Department.** K. MARTINEZ. Aero Digest **36,** 26–28 (1940).

10. Supersonics (Ultrasonics)

10.1 **Supersonic Analogy of Electromagnetic Field.** D. Riabouchinsky. Comptes Rendus **209,** 664–666 (1939).

10.2 **Optical Method for Measuring Supersonic Absorption in Gases and Liquids.** O. Petersen. Phys. Zeits. **41,** 29–36 (1940). Sci. Abs. **A43,** 1896 (1940).

10.2 **Diffraction of Light by Supersonic Waves in Liquids.** A. W. Smith and L. M. Ewing. Am. J. Phys. **8,** 57–59 (1940). Sci. Abs. **A43,** 1623 (1940).

10.3 **A Constant-Path Acoustic Interferometer for Gases at Variable Pressure.** C. M. Herget. Rev. Sci. Instruments **11,** 37–39 (1940).

10.3 **Fixed Path Acoustic Interferometer for the Study of Matter.** J. C. Hubbard and I. F. Zartman. Rev. Sci. Instruments **10,** 382–386 (1939).

10.4 **Theory of Measurement of Absorption of Supersonics.** E. David. Phys. Zeits. **41,** 37–41 (1940). Sci. Abs. **A43,** 1897 (1940).

10.4 **Specific Heats and Dispersion of Supersonic Velocity in Organic Vapours.** S. K. K. Jatkar. Indian Journ. Phys. **13,** 445–449 (1939). Sci. Abs. **A43,** 1895 (1940).

10.4 **Supersonic Velocities in Gases and Vapours.** S. K. K. Jatkar. Indian Journ. Phys. **13,** 445–449 (1939).

10.4 **Origin of Phase Difference between Pressure- and Density Waves during Molecular Absorption of Sound in Gases. Simple Derivation of Dispersion and Absorption Formulae.** O. Numoto. Phys. Math. Soc. Japan, Proc. **22,** 77–90 (1940). Sci. Abs. **A43,** 1890 (1940).

10.4 **Optical Method for Measuring Supersonic Absorption in Gases and Liquids.** O. Petersen. Phys. Zeits. **41,** 29–36 (1940). Sci. Abs. **A43,** 1896 (1940).

10.5 **Supersonic Velocities and Adiabatic Compressibilities of Organic Liquids.** J. Bhimasenachar and K. Venkateswarden. Indian Acad. Sci., Proc. Section A **11,** 28–31 (1940). Sci. Abs. **A43,** 1237 (1940).

10.5 **The Propagation of Supersonics in Liquids.** E. G. Richardson. Phys. Soc., Proc. **52,** 480–88 (1940).

10.8 **Chemical Effects of Ultrasonic Waves.** W. Demann and H. R. Asbach. Technische Metterlungen. Krupp Forschungsberichte **3,** 12–25 (1940). Chem. Abs. **34,** 3972 (1940).

11. Waves and Vibrations

11.1 **Observations on Edge Tones.** J. M. A. Lenihan and E. G. Richardson. Phil. Mag. **29,** 400–406 (1940). Sci. Abs. **A43,** 1621 (1940).

11.1 **Sound.** E. G. Richardson. Phys. Soc., Reports **6,** 238–269 (1939).

11.5 **Über das Ersatzschema des Kugelknalls.** L. Cremer. Akustische Zeits. **5,** 46 (1940).

11.5 **Eine Methode zur Erzeugung Extrem Starker Stehender Schallwellen in Luft. (A Method of Producing Extremely Strong Standing Sound Waves in Air.)** Hermann Oberst. Akustische Zeits. **5,** 27 (1940).

11.7 **Der Fehler bei der Lichtbandbreitenmessung an Schallplatten.** Richard Bierl. Akustische Zeits. **5,** 145 (1940).

11.7 **Form and Propagation of Sound Waves in a Space Limited by Absorbing Surfaces.** J. Brillouin. J. de Physique et le Radium **10,** 497–508 (1939). Sci. Abs. **A43,** 1230 (1940).

11.7 **Inertia of the Regimes in Oscillating Fluids.** Z. Carrière. J. de Physique et le Radium **11,** 68–73 (1940). Sci. Abs. **A43,** 1620 (1940).

11.7 **Asymptotic Distribution of Frequencies of a Hollow Space and Surface Tension of an Ideal Gas.** K. Husini. Phys. Math. Soc. Japan, Proc. **21,** 759–768 (1939). Sci. Abs. **A43,** 1889 (1940).

11.7 **The Absorption of Sound by Systems with Double Resonance.** V. S. Nesterov. J. Techn. Physics, U. S. S. R. **9,** 1727–1739 (1939). (In Russian.)

11.7 **Experimentelle Untersuchung eines Quasiharmonischen Schwingers. (Experimental Research on Quasi-Harmonic Vibrators.)** Hanns Neusinger. Akustische Zeits. **5,** 11 (1940).

11.7 **Vibration of Metallic Discs by Rhythmic Sublimation.** T. Okaya and T. Töbusi. Phys. Math. Soc. Japan, Proc. **21,** 768–786 (1939). Sci. Abs. **A43,** 1891 (1940).

11.7 **Supersonic Analogy of Electromagnetic Field.** D. Riabouchinsky. Comptes Rendus **209,** 664–666 (1939). Sci. Abs. **A43,** 1236 (1940).

11.7 **Acoustic Radiation Pressure.** G. Richter. Zeits. f. Physik **115,** 97–100 (1940).

11.7 **Density of Energy in Acoustic Problems.** G. Sacerdote. Pont. Acad. Sci. Acta **3,** 47–52 (1939). Sci. Abs. **A43,** 1619 (1940).